

Australian Standard™

Design of rotating steel shafts

This Australian Standard was prepared by Committee ME-005, Cranes. It was approved on behalf of the Council of Standards Australia on 9 March 2004 and published on 21 May 2004.

The following are represented on Committee ME-005:

- Australian Chamber of Commerce and Industry
- Australian Elevator Association
- Australian Industry Group
- Australian Institute for Non-Destructive Testing
- Bureau of Steel Manufacturers of Australia
- Construction and Mining Equipment Association of Australia
- Crane Industry Council of Australia
- Department of Administrative and Information Services (South Australia)
- Department of Employment Training and Industrial Relations (Qld)
- Department of Infrastructure, Energy and Resources (Tasmania)
- Department of Labour New Zealand
- Institution of Engineers Australia
- State Chamber of Commerce
- The Association of Consulting Engineers Australia
- The University of New South Wales
- Victorian WorkCover Authority
- WorkCover NSW
- WorkSafe Western Australia

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Design of rotating steel shafts

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PREFACE

This Standard was prepared by the Joint Standards Australia/Standards New Zealand Committee ME-005, Cranes, to supersede AS 1403—1985.

The 1985 edition was reviewed by the Committee, and it was found to be technically valid. Hence, this edition does not include technical alterations to the 1985 edition; however, the referenced documents have been updated, and editorial changes have been made to align with current Standards Australia policy.

The term 'informative' has been used in this Standard to define the application of the appendix to which it applies. An informative appendix is only for information and guidance.

CONTENTS

	<i>Page</i>
FOREWORD.....	4
1 SCOPE.....	5
2 REFERENCED DOCUMENTS.....	5
3 DEFINITIONS.....	6
4 NOTATION.....	6
5 APPLIED LOADING	8
6 MINIMUM CALCULATED DIAMETER OF SHAFT	8
7 SHAFT DESIGN AND MANUFACTURE.....	9
8 SHAFT DESIGN FACTORS.....	10
APPENDICES	
A 'TRIAL' SHAFT DIAMETER	21
B ITERATIVE METHOD FOR CALCULATING MINIMUM DIAMETER OF SHAFT	22
C CHARACTERISTICS OF MOTOR CONTROLLERS AND TORQUE-LIMITING DEVICES	23
D TYPICAL WORKED EXAMPLE—CRANE DRIVING MECHANISM.....	26
E TYPICAL WORKED EXAMPLE—CRANE HOIST DRIVE.....	36
F TYPICAL WORKED EXAMPLE—CONVEYOR DRIVE.....	45

FOREWORD

The method of design in this Standard is the same as that contained in the 1984 issue and is based on calculations for infinite life of the shaft.

It is noted that, as the result of extensive laboratory and/or field tests, shafts of diameters smaller than those resulting from calculations in this Standard may be used.

Although the method is not theoretically precise, it gives results that are of sufficient accuracy for practical purposes. The method makes no allowance for corrosive or other abnormal conditions, such as the presence of buckling, 'whipping', cyclic vibrations and similar effects (see Clause 7.2).

In some cases, deflection may be the factor that determines the minimum value of the shaft diameter (see Clause 7.6).

Typical worked examples employing this method of shaft design are given in Appendices D, E and F.

A 'trial' diameter may be necessary in the application of the design method. The value may be assumed on the basis of the designer's previous experience, or a simplified version of the method may be used to determine the order of magnitude of the shaft diameter, and the full method then applied to check the assumed trial diameter. A convenient quick method of estimating the 'trial' diameter is given in Appendix A, and an iterative method using programmable calculators is provided in Appendix B.

The process of carrying out an accurate and complete analysis of loadings applied to a shaft (see Clause 5) may involve much time and effort, and is warranted where it is desired to keep shaft diameters as small as possible. Where larger diameter shafts can be tolerated, it may be more appropriate to use approximations or to ignore inertia or other effects, but it is important to ensure that, where such inaccuracies are introduced, they result in increased rather than decreased margin of safety and stress-raising characteristics are avoided.

The effect of electric motors with high breakdown torque (also known as pull-out torque) or high locked-rotor torque (also known as starting torque) and the characteristics of motor controllers and torque-limiting devices are outlined in Appendix C.

STANDARDS AUSTRALIA

Australian Standard Design of rotating steel shafts

1 SCOPE

This Standard provides formulae for the design of rotating steel shafts that are subjected to torsional, bending and axial-tensile loads either singly or in combination, on the basis of infinite life.

The Standard does not cover specially developed shafts, for example, those involving extensive laboratory and field testing, heat treatment and like developments, or to shafts for specific applications such as automotive and construction equipment transmissions.

NOTES:

- 1 Particular consideration has to be given to deflections of long shafts where the effects of out-of-balance forces tend to induce 'whipping', and to long shafts that are subject to buckling due to compressive loading (see Foreword).
- 2 The use of this method of shaft design enables justification of the use of shafts of minimal diameter. Where the designer conservatively sizes a shaft, then only those calculations necessary to ensure that the shaft complies with this Standard (see Clause 5) are required.
- 3 A 'trial' shaft diameter is given in Appendix A.
- 4 An interactive method for calculating minimum diameter of shaft is given in Appendix B.
- 5 Characteristics of motor controllers and torque-limiting devices are provided in Appendix C.
- 6 A typical worked example for crane driving mechanism is provided in Appendix D.
- 7 A typical worked example for crane hoist drive is provided in Appendix E.
- 8 A typical worked example for conveyor drive is provided in Appendix F.

2 REFERENCED DOCUMENTS

The following documents are referred to in this standard:

AS

- | | |
|--------|---|
| 1391 | Methods for tensile testing of metals |
| 1654 | ISO system for limits and fits |
| 1654.1 | Part 1: Bases of tolerances, deviations and fits |
| 1654.2 | Part 2: Tables of standard tolerance grades and limit deviations for holes and shafts |

ISO

- | | |
|------|--|
| 14 | Straight-sided splines for cylindrical shafts with internal centering; Dimensions, tolerances and verification |
| 4156 | Straight cylindrical involute splines; Metric module, side fit; Generalities, dimensions and inspection |

BS

- | | |
|------|---|
| 2059 | Specification for straight-sided splines and serrations |
|------|---|

3 DEFINITIONS

For the purpose of this Standard, the definitions below apply.

3.1 Applied loading

3.1.1 Axial loading

Force applied to a shaft in the direction of the shaft axis.

3.1.2 Bending loading

Bending moment applied to a shaft.

3.1.3 Torsional loading

Torque applied to a shaft.

3.2 Equivalent rotational mass moment of inertia of a shaft and its associated components

The linear mass moment of inertia and the rotational mass moment of inertia of the component, related to the rotational full-load speed of the braking or driving medium of the mechanism.

3.3 Stress-raising characteristic

A detail, of a shaft, that induces increased stress in the shaft, e.g. step in diameter, keyway, spline, interference fit with a component on the shaft, transverse hole, annular groove.

NOTE: Stress-raising effects are intensified by machining marks, notching and the like.

4 NOTATION

For the purpose of this Standard, the following notation applies:

D	=	minimum calculated diameter of shaft at cross-section under consideration, in millimetres (see also Clause 6.2)
D_a	=	estimated diameter in iterative method (see Appendix B)
D_{a-1}	=	previous estimated diameter in iterative method (see Appendix B)
D_i	=	internal diameter of hollow shaft, in millimetres
D_o	=	minimum calculated outside diameter of hollow shaft at cross-section under consideration, in millimetres
e	=	allowable deviation between successive iterations, expressed as a percentage (see Appendix B, Figure B1)
F_R	=	endurance limit of shaft material in reversed bending during rotation, based on tests of polished steel specimens of diameter between 8 mm and 10 mm, in megapascals
	=	$0.45 F_U$, where actual value is not known
F_S	=	safety factor
F_U	=	tensile strength of shaft material, in megapascals
F_Y	=	yield strength of shaft material, in megapascals
I_{Ei}	=	equivalent rotational mass moment of inertia of a shaft and its associated rotating and linear components, in kilogram metres squared
	=	$I_{Ri} + I_{Lj}$
I_{Ri}	=	equivalent rotational mass moment of inertia of a shaft and its associated rotating components, in kilogram metres squared

$$= I_i \left(\frac{N_i}{N} \right)^2$$

I_{Lj} = equivalent rotational mass moment of inertia of components with linear motion, associated with the shaft under consideration, in kilogram metres squared

$$= m_j \left(\frac{V_j}{2\pi N} \right)^2$$

- $I_1, I_2, I_3 \dots I_n$ = rotational mass moment of inertia at first, second, third ... n th shafts and its components rotating at $N_1, N_2, N_3 \dots N_n$ revolutions per second, in kilogram metres squared
- i = number defining the rotational motion component under consideration (1, 2, 3, ... n)
- j = number defining the linear motion component under consideration (1, 2, 3 ... k)
- k = total number of components with linear motion
- K = stress-raising factor (see Clause 8.2):
for a stepped shaft, see Figure 4
for a shaft fitted with rolling-element bearing, see Figure 5
for a shaft with fitted component without key or spline, see Figure 6
for a shaft with keyed component, see Figure 7
for a splined shaft, see Figure 8
for a shaft with annular groove, see Figure 9
for a shaft with transverse hole, see Figure 10
- K_s = size factor (see Clause 8.1 and Figure 1)
- m_j = mass of linear-motion component, in kilograms
- M_q = bending moment at shaft cross-section under consideration, in newton metres
- n = total number of components with rotational motion
- N = rotational full-load speed of driving or braking medium, in revolutions per second
- N_i = rotational full-load speed of a shaft and its components, in revolutions per second
- $N_1, N_2, N_3 \dots N_n$ = rotational full-load speeds of 1st, 2nd, 3rd .. n th rotating elements in gear train or other speed-changing device, in revolutions per second
- P_q = maximum axial tensile force at shaft cross-section under consideration, in newtons
- NOTE: For axial compressive force, see Clause 7.2.
- S_0 = original cross-sectional area within the gauge length (see Clause 7.1)
- T_E = equivalent torque when calculating 'trial' diameter, in newton metres (see Appendix A)
- T_q = maximum torque at shaft cross-section under consideration, in newton metres
- T_M = torque applied to the mechanism by the braking or driving means or by the external load, in newton metres
- V_j = velocity of linear-motion component, in metres per second

α	=	angular acceleration of driving medium or braking medium, in radians per second squared
$\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$	=	angular acceleration of the first, second, third ... <i>n</i> th shafts of a gear train or other speed changing device, in radians per second squared
Δ	=	correction factor (see Figure 3)
η	=	mechanical efficiency of system
$\eta_{i/i+1}$	=	mechanical efficiency of mating components
$\eta_{1/2}, \eta_{2/3}, \eta_{3/4}, \dots, \eta_{n-1/n}$	=	mechanical efficiency of gear pair or other speed changing device between first/second, second/third, third/fourth ... (<i>n</i> – 1)th/ <i>n</i> th shafts

5 APPLIED LOADING

Determination of the axial, bending and torsional loading (see Clause 3.1) applied to a component shaft of a mechanism shall take into account the following effects:

- Torque applied by the driving or braking medium.
- Reactive torque of the element driven by the mechanism including, where applicable, dynamic effects.
- Energy gain and loss during acceleration and deceleration of the system.
- Mechanical efficiency.

NOTES:

- The process of completely and accurately analysing the loading applied to a component shaft of a mechanism may involve considerable time and effort (except where performed by computer with existing programs) and is necessary where it is desired to keep shaft diameter as small as possible.

Where the size of a shaft diameter is not critical or needs to be relatively large to provide adequate stiffness (see Clause 7.6) or to accommodate a component, experience may indicate that a shaft size selected need not be checked by calculation or may be checked by use of approximations or by ignoring mechanical efficiency, inertia and like effects. It is essential, however, that under such circumstances the margin of safety applicable to the shaft is increased rather than decreased.

- It may be more accurate and easier to calculate the loading of a particular component shaft of a mechanism from either the driving end or the driven end of the system depending on the characteristics of the system and the driven element.
- The presence and location of a flywheel in the system should be taken into account in the selection of the appropriate formula from Table 1, e.g. flywheel driving effect during braking.

The theoretical value of the torque applied to a component shaft of a gear train or other speed-changing device shall be not less than the value calculated by means of the appropriate formulas given in Table 1.

6 MINIMUM CALCULATED DIAMETER OF SHAFT

6.1 Minimum calculated diameter

The minimum calculated diameter of each stressed section of shaft shall be calculated by means of the appropriate formula given in Table 2.

6.2 Solid shafts

The minimum calculated diameter for solid shafts shall comply with the following requirements:

- (a) Solid shafts without keyways or with maximum of two keyways. The outside diameter shall be not less than D .

NOTE: The influence of the keyway(s) is taken into account by Figure 7.

- (b) Solid shafts with splines. The root diameter of the splines shall be not less than D .

NOTE: The influence of the splines is taken into account by Figure 8.

- (c) Solid shafts with annular grooves. The diameter at the base of the groove shall be not less than D .

NOTE: The influence of the groove is taken into account by Figure 9.

6.3 Hollow shafts

Hollow shafts shall comply with the following requirements:

- (a) The shaft shall have a constant diameter longitudinal hole, coaxial with the outside diameter.
- (b) The wall thickness shall be greater than $0.15 D$.
- (c) The outside diameter shall be greater than the value of D_o calculated by the following formula:

$$D_o^3 = D^3 + 1.7 D_i^3 \quad \dots 5$$

7 SHAFT DESIGN AND MANUFACTURE

7.1 Shaft material

The steel from which the shaft is manufactured shall have an elongation of not less than 5% on a gauge length of $5.65\sqrt{S_o}$ when tested in accordance with AS 1391.

NOTE: Where the shaft is manufactured from a high strength steel, fatigue strength is generally affected more critically by stress-raising characteristics of the shaft than where the shaft is manufactured from a low strength steel.

7.2 Shaft diameters

The required diameter for each stressed section of the shaft shall be specified in accordance with Clause 6.1.

Where shafts are to operate under indeterminate or particularly arduous and abnormal conditions, and such aspects as corrosion fatigue, fretting fatigue or high temperature influences and the like may be involved, additional allowances shall be made for the deleterious effects of such conditions by making the actual diameter greater than the theoretical size.

Where a shaft is subjected to an axial compression force, the value of P_q is taken as 0 (zero), but if buckling, 'whipping', cyclic vibrations and like effects exist, it is essential that provision be made to ensure that—

- (a) the shaft diameter is adequate to withstand any additional stresses due to such effects; and
- (b) such effects are limited to the extent that the operation of the mechanism in which the shaft is incorporated is not adversely affected.

7.3 Stress-raising characteristics (see Clause 8.2)

The shaft shall be designed to minimize the effects of stress-raising characteristics. Special care shall be taken in the manufacture to achieve a smooth finish of highly stressed portions; this shall apply particularly where stress concentrations occur, e.g., at changes of shaft diameters. Radii at steps in diameter and at the bottoms of keyways shall be as large as practicable and large changes in diameter should be avoided.

7.4 Welding on shafts

Shafts for transmission of power shall not have welds in any position where the tensile stress exceeds 15 MPa and the number of stress cycles exceeds 1000 in the design life of the shaft, unless the designer makes adequate provision for—

- (a) the weld shape and quality in respect of the stress-raising effect of the weld;
- (b) the reduced fatigue performance of welded higher strength steels; and
- (c) the possible reduced mechanical properties of the shaft in the weld.

NOTE: The performance of shafts can be seriously impaired by welding. The design of shafts containing welds is not covered by this Standard.

7.5 Splines and keyways

Splines and keyways shall conform to one of the relevant standards, i.e., ISO 14, ISO 4156 and BS 2059.

7.6 Deflection

The deflection of the shaft under operating conditions shall not be greater than the values dictated by gear-separation tolerances, bearing manufacturers' recommendations, locking elements and like effects (see Foreword).

8 SHAFT DESIGN FACTORS

8.1 Size factor (K_s)

The value of the size factor (K_s) shall be as follows:

- (a) For shaft diameters up to 250 mm the value in Figure 1 that corresponds to the actual diameter of the stressed section of the shaft.
- (b) For shaft diameters greater than 250 mm 1.8.

NOTE: When the shaft diameter is not known, a 'trial' diameter (see Appendices A and B) should be calculated and the factor K_s selected accordingly.

8.2 Stress-raising factor (K)

The value of the stress-raising factor (K) shall be as follows:

- (a) Where there is only one stress-raising characteristic the value read from Figures 4 to 10, as appropriate.
- (b) Where two stress-raising characteristics are separated by an axial distance greater than $0.25D$ the greater of the two values read from Figures 4 to 10, as appropriate (see Figure 2).
- (c) Where two stress-raising characteristics are separated by an axial distance between $0.16D$ and $0.25D$ the sum of the greater value and 0.1 times the lesser value (see Figure 2), both values being those read from Figures 4 to 10, as appropriate.
- (d) Where two stress-raising characteristics are coincident or separated by an axial distance not greater than $0.16D$ the sum of the greater value and 0.2 times the lesser value (see Figure 2) both values being those read from Figures 4 to 10, as appropriate.

NOTE: When the shaft diameter is not known, a 'trial' diameter (see Appendices A and B) should be calculated and the factor K selected accordingly.

TABLE 1
FORMULAS FOR SHAFT TORQUE T_q

System	Shaft	Torque	Inertia significant	Inertia not significant
Driving/Accelerating	1	Input	T_M	T_M
		Output	$T_M - I_{E1} \alpha$	
	2	Input	$(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \eta_{1/2}$	$T_M \frac{N_1}{N_2} \eta_{1/2}$
		Output	$(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \eta_{1/2} - I_{E2} \frac{N_1}{N_2} \alpha$	
	3	Input	$\left[(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \eta_{1/2} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \eta_{2/3}$	$T_M \frac{N_1}{N_2} \eta_{1/2} \eta_{2/3}$
		Output	$\left[(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \eta_{1/2} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \eta_{2/3} - I_{E3} \frac{N_1}{N_3} \alpha$	
	4	Input	$\left\{ \left[(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \eta_{1/2} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \eta_{2/3} - I_{E3} \frac{N_1}{N_3} \alpha \right\} \frac{N_3}{N_4} \eta_{3/4}$	$T_M \frac{N_1}{N_2} \eta_{1/2} \eta_{2/3} \eta_{3/4}$
		Output	$\left\{ \left[(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \eta_{1/2} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \eta_{2/3} - I_{E3} \frac{N_1}{N_3} \alpha \right\} \frac{N_3}{N_4} \eta_{3/4} - I_{E4} \frac{N_1}{N_4} \alpha$	
Braking/Decelerating	1	Input	T_M	T_M
		Output	$T_M - I_{E1} \alpha$	
	2	Input	$(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \frac{1}{\eta_{1/2}}$	$T_M \frac{N_1}{N_2} \frac{1}{\eta_{1/2}}$
		Output	$(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \frac{1}{\eta_{1/2}} - I_{E2} \frac{N_1}{N_2} \alpha$	
	3	Input	$\left[(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \frac{1}{\eta_{1/2}} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \frac{1}{\eta_{2/3}}$	$T_M \frac{N_1}{N_3} \frac{1}{\eta_{1/2} \eta_{2/3}}$
		Output	$\left[(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \frac{1}{\eta_{1/2}} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \frac{1}{\eta_{2/3}} - I_{E3} \frac{N_1}{N_3} \alpha$	
	4	Input	$\left\{ \left[(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \frac{1}{\eta_{1/2}} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \frac{1}{\eta_{2/3}} - I_{E3} \frac{N_1}{N_3} \alpha \right\} \frac{N_3}{N_4} \frac{1}{\eta_{3/4}}$	$T_M \frac{N_1}{N_4} \frac{1}{\eta_{1/2} \eta_{2/3} \eta_{3/4}}$
		Output	$\left\{ \left[(T_M - I_{E1} \alpha) \frac{N_1}{N_2} \frac{1}{\eta_{1/2}} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \frac{1}{\eta_{2/3}} - I_{E3} \frac{N_1}{N_3} \alpha \right\} \frac{N_3}{N_4} \frac{1}{\eta_{3/4}} - I_{E4} \frac{N_1}{N_4} \alpha$	

TABLE 2
FORMULAS FOR CALCULATING MINIMUM DIAMETER OF SHAFT D

Number of mechanism starts per year	Number of revolutions of shaft per year	Torque application conditions	Formula	Formulas
≤600	≤900	Manually or power applied	$D^3 = \frac{10^4 F_S}{F_Y} \sqrt{\left(M_q + \frac{P_q D}{8000} \right)^2 + \frac{3}{4} T_q^2}$	1
	>900	Power applied	$D^3 = \frac{10^4 F_S}{F_R} \sqrt{\left[K_S K \left(M_q + \frac{P_q D}{8000} \right) \right]^2 + \frac{3}{4} T_q^2}$	2
>600	>900	Power applied torque reversals	$D^3 = \frac{10^4 F_S}{F_R} K_S K \sqrt{\left(M_q + \frac{P_q D}{8000} \right)^2 + \frac{3}{4} T_q^2}$	3
		Power applied, no torque reversals (see Note 1)	$D^3 = \frac{10^4 F_S}{F_R} \sqrt{\left[K_S K \left(M_q + \frac{P_q D}{8000} \right) \right]^2 + \frac{3}{16} \left[(1 + K_S K) T_q \right]^2}$	4

NOTES:

- Where the magnitude of the torque in one direction is not greater than 0.1 times the torque in the other direction, the torque application conditions may be considered as being non-reversing.
- These formulas are extracted from a paper titled 'Shortcuts for Designing Shafts' by H.A. Borchardt, published in *Machine Design*, Vol. 45, No.3, 8 February 1973, pp 139-141.
- The value of F_R is based on 10^6 stress cycles and is applicable for numbers of revolutions of shafts per year greater than 50 000.
- For numbers of revolutions of shafts per year from 50 000 down to 900, formulas 2, 3 and 4 result in progressively more conservative values for the theoretical diameter of shaft.
- The value of F_S is 2.0 for Formula 1 and 1.2 for Formulas 2, 3 and 4. Where severe injury, death or extensive equipment damage is likely to occur because of the failure of the shaft, higher factors of safety may be used.
- The values of K_S , K and the term $P_q D/8000$ may require the calculation of a 'trial' diameter (see Appendix A).

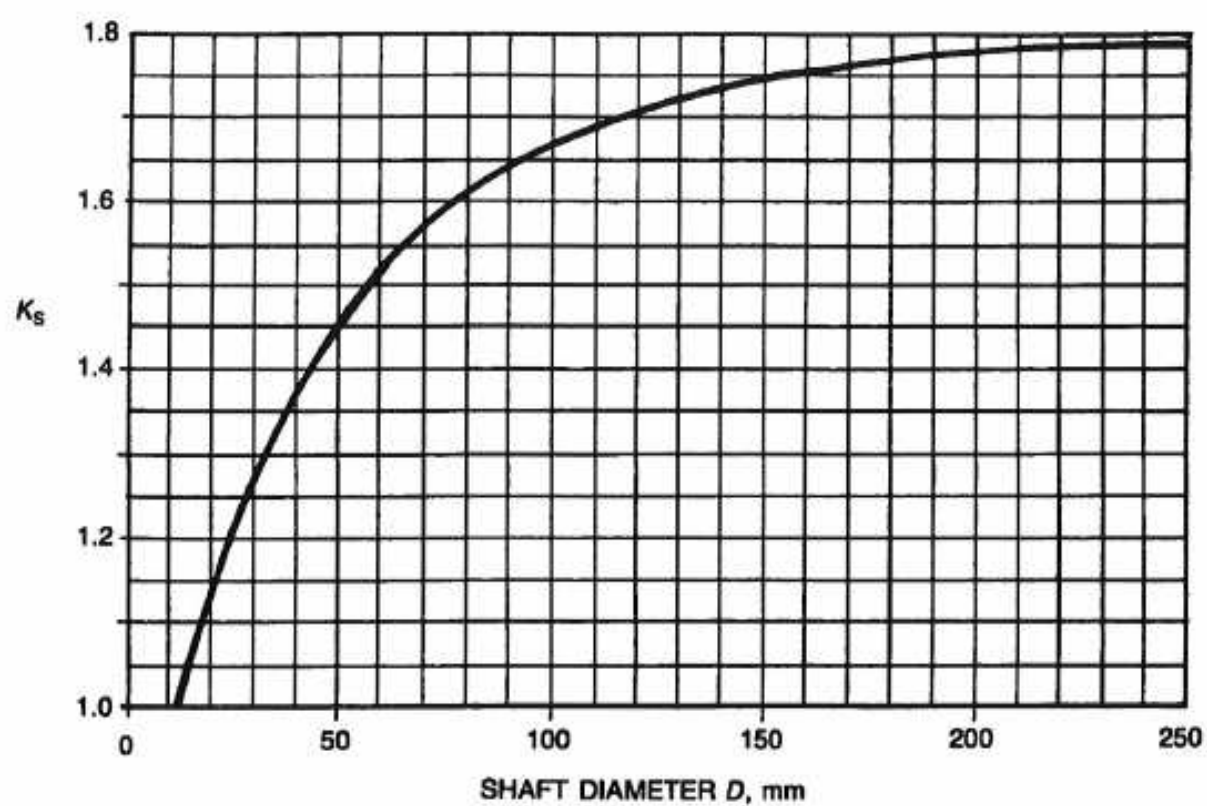


FIGURE 1 SIZE FACTOR K_s

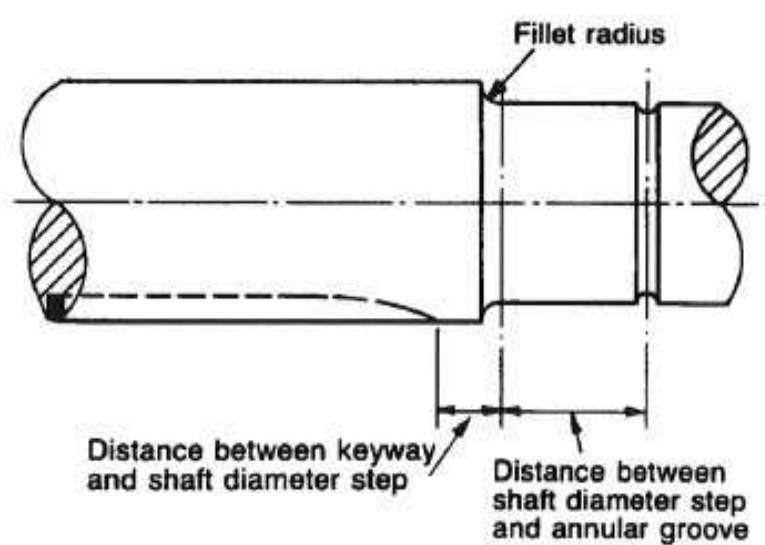
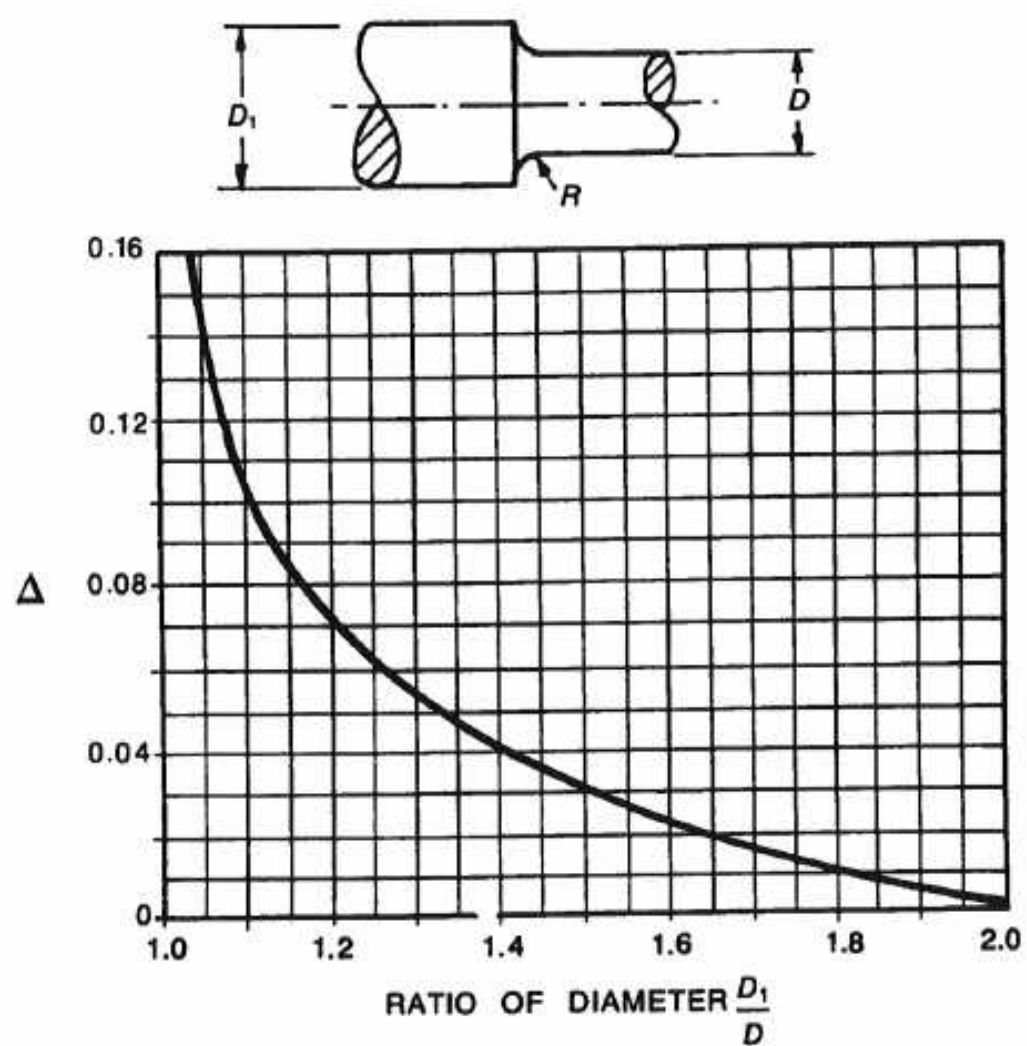


FIGURE 2 TYPICAL EXAMPLES OF STRESS-RAISING CHARACTERISTICS

FIGURE 3 CORRECTION FACTOR Δ

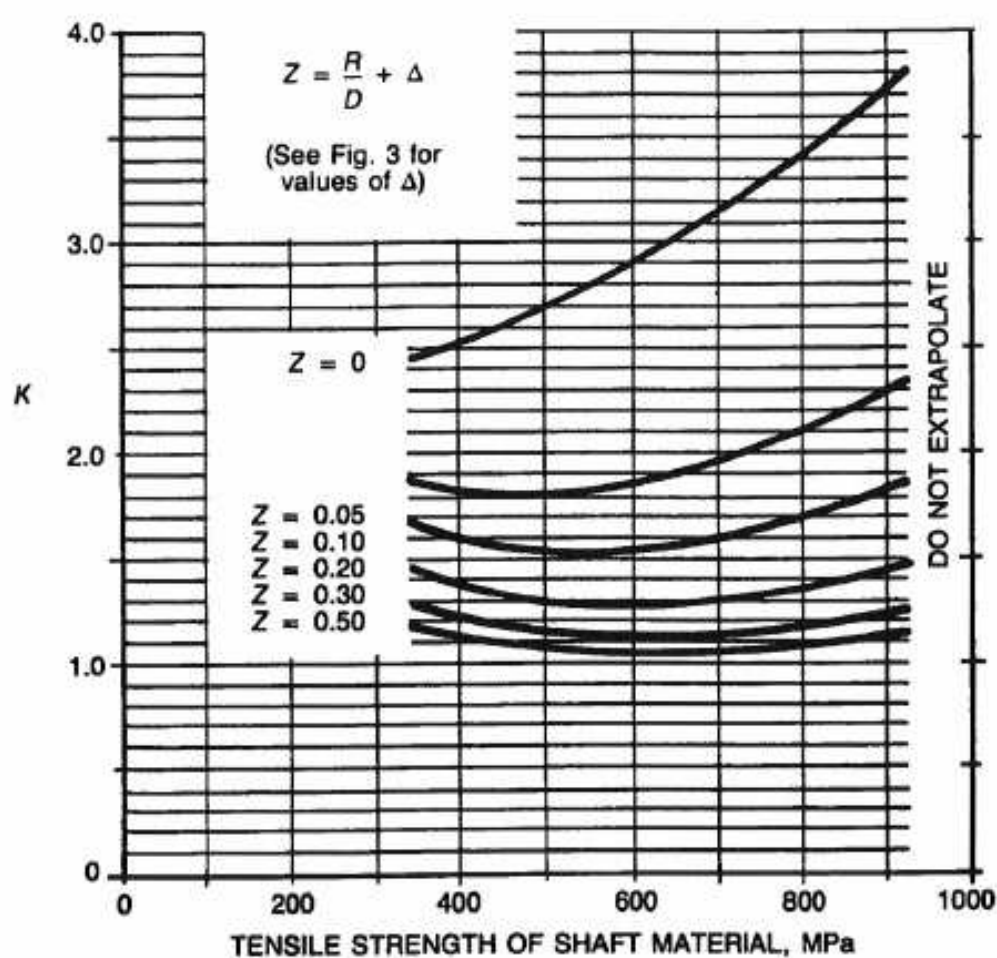
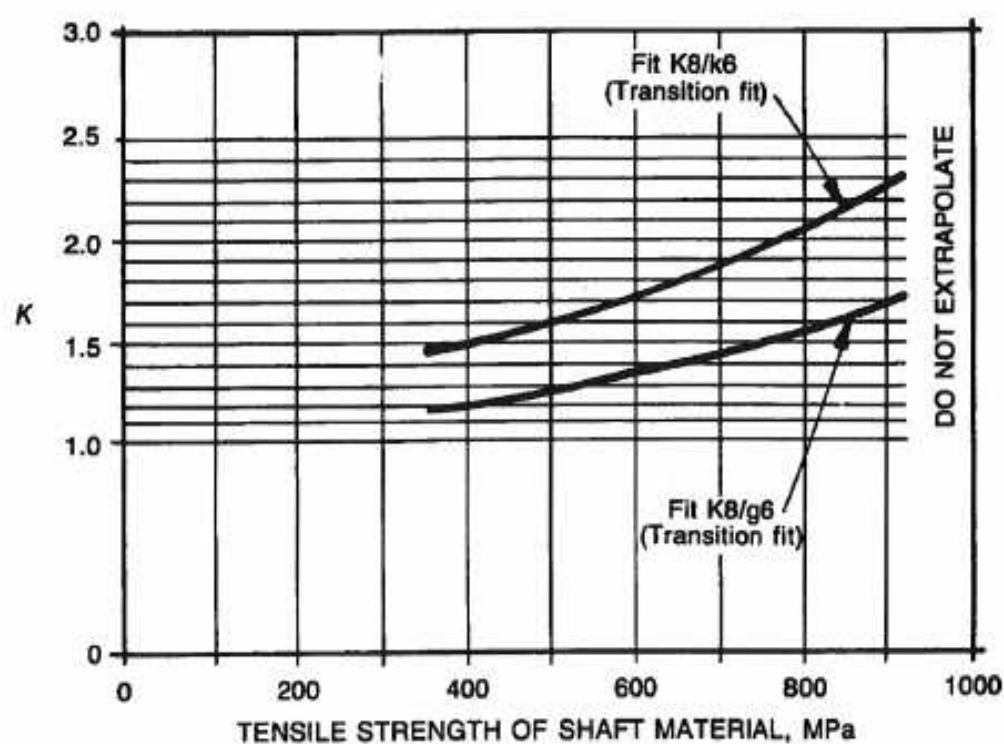


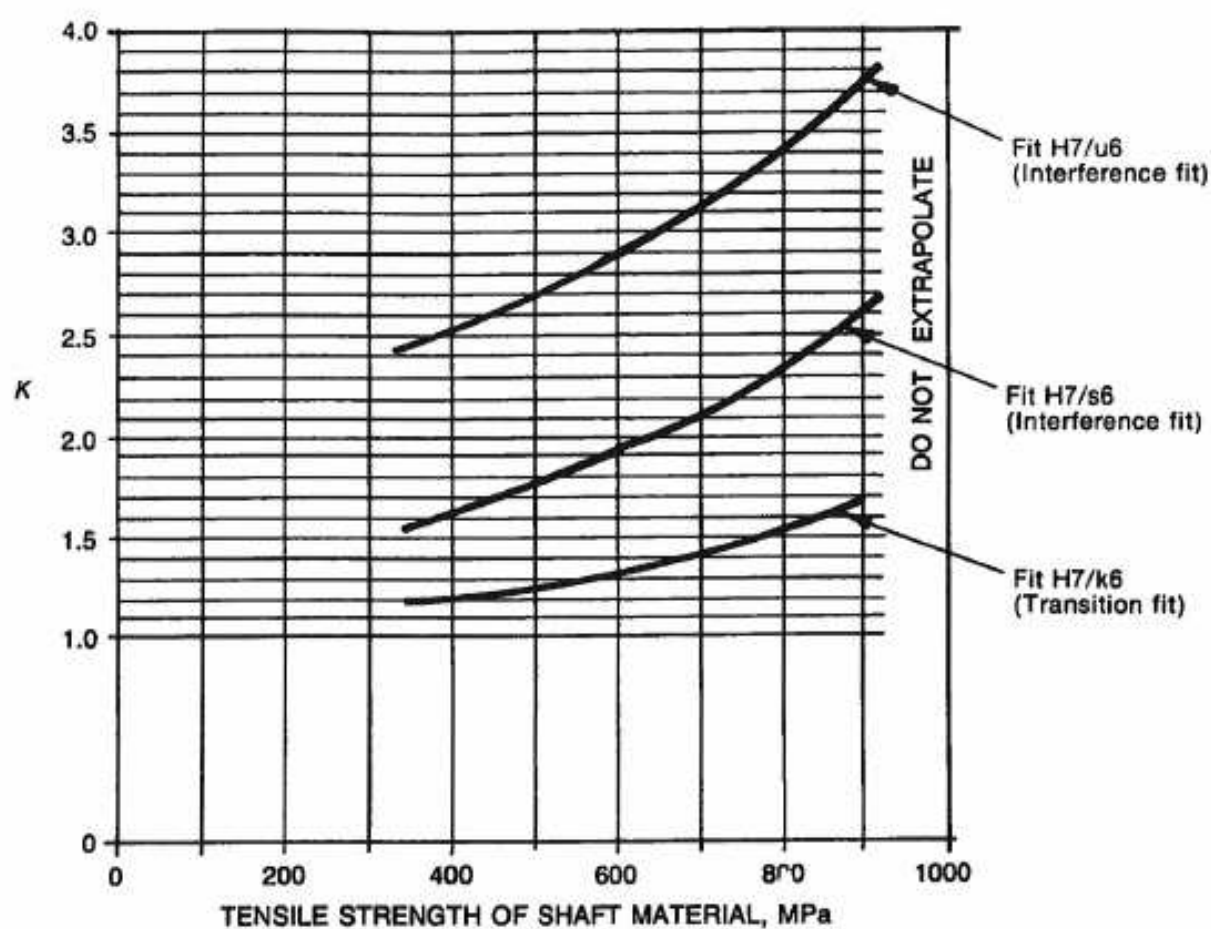
FIGURE 4 STRESS-RAISING FACTOR K FOR STEPPED SHAFT



NOTES:

- 1 Values may be interpolated for fits between K8/k6 and K8/g6 which are recommended by the bearing manufacturers.
- 2 For tolerances, see AS 1654.

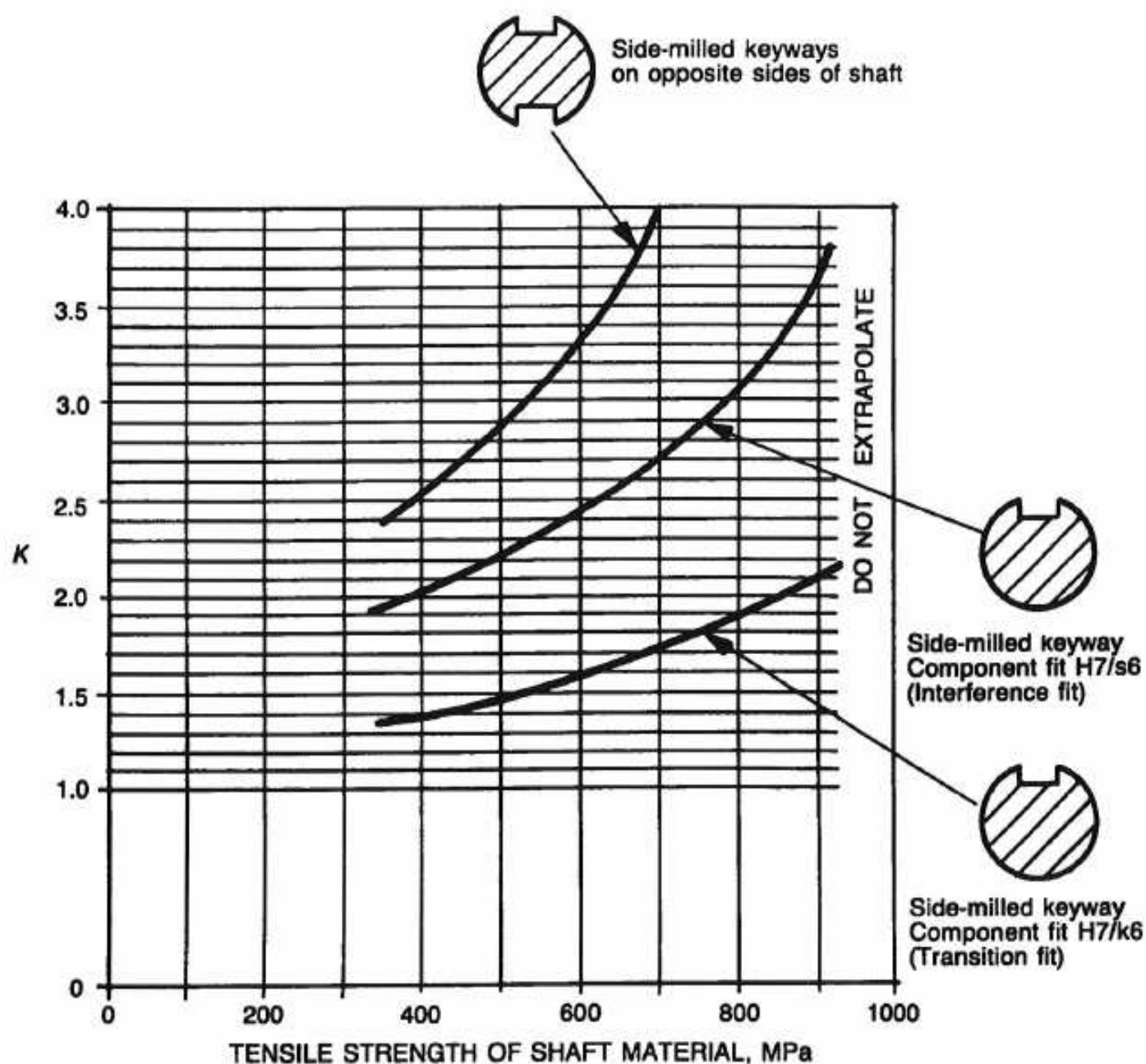
FIGURE 5 STRESS-RAISING FACTOR K FOR SHAFT FITTED WITH ROLLING ELEMENT BEARING



NOTES:

- 1 Values may be interpolated for other fits between H7/u6 and H7/k6.
- 2 For tolerances, see AS 1654.
- 3 For stress-raising factors of locking devices refer to the manufacturer's recommendations.

FIGURE 6 STRESS-RAISING FACTOR K FOR FITTED COMPONENT WITHOUT KEY OR SPLINE



NOTES:

- 1 For end-milled keyway with blind-end, multiply factor K by 1.1.
- 2 Values may be interpolated for fits between H7/s6 and H7/k6.
- 3 For tolerances, see AS 1654.

FIGURE 7 STRESS-RAISING FACTOR K FOR KEYED COMPONENT

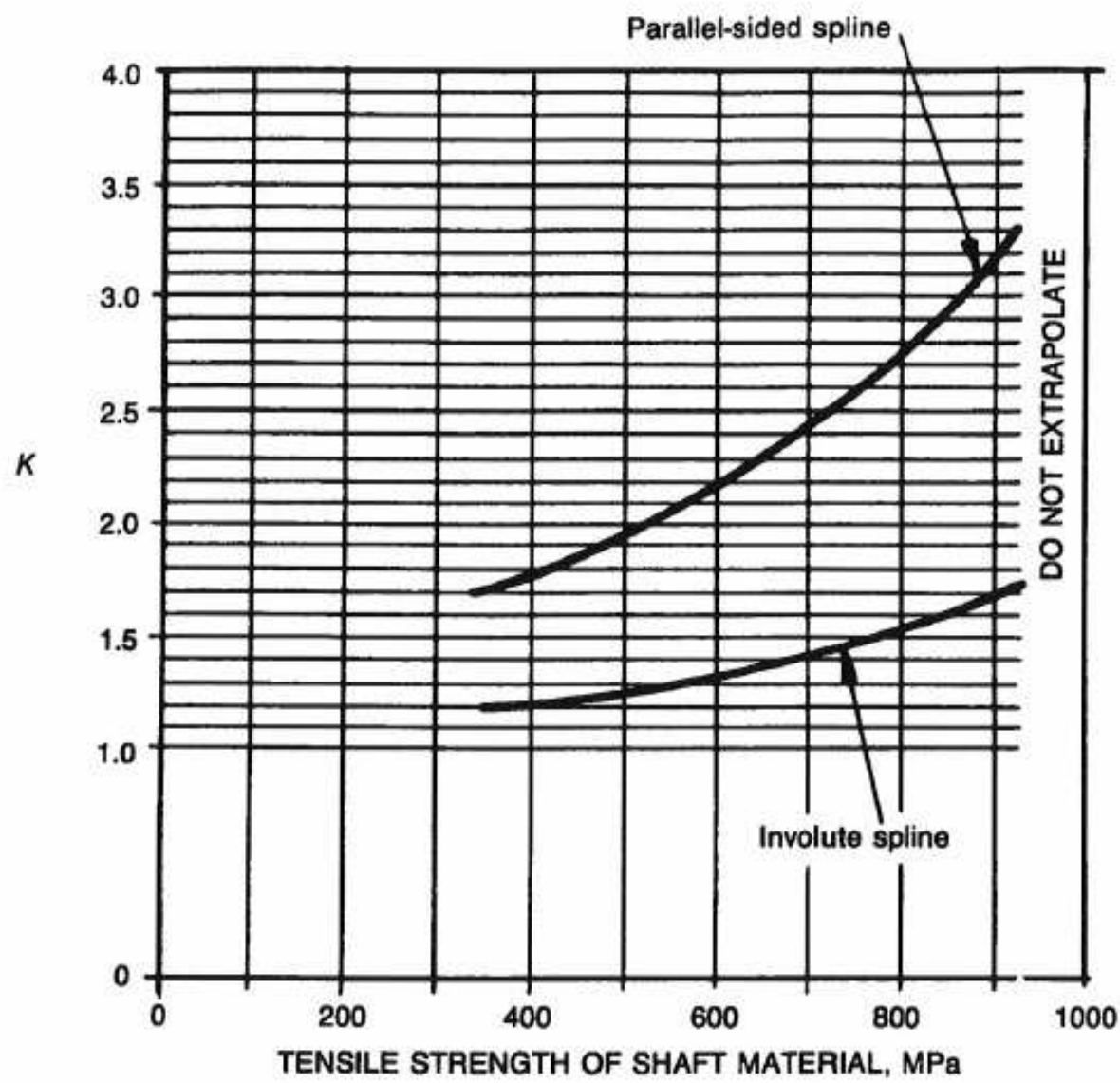


FIGURE 8 STRESS-RAISING FACTOR K FOR SPLINED SHAFT

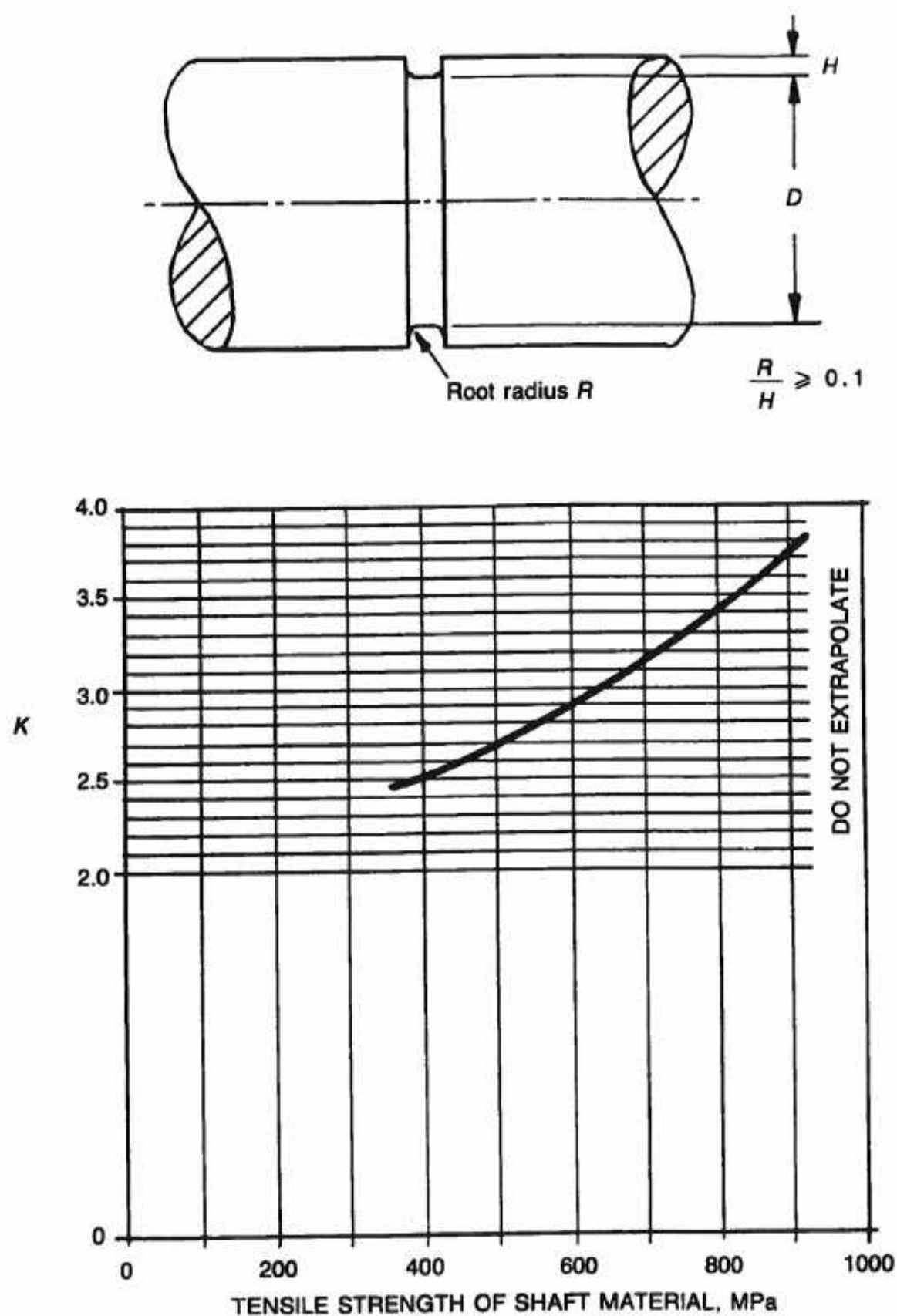


FIGURE 9 STRESS-RAISING FACTOR K FOR SHAFT WITH ANNULAR GROOVE

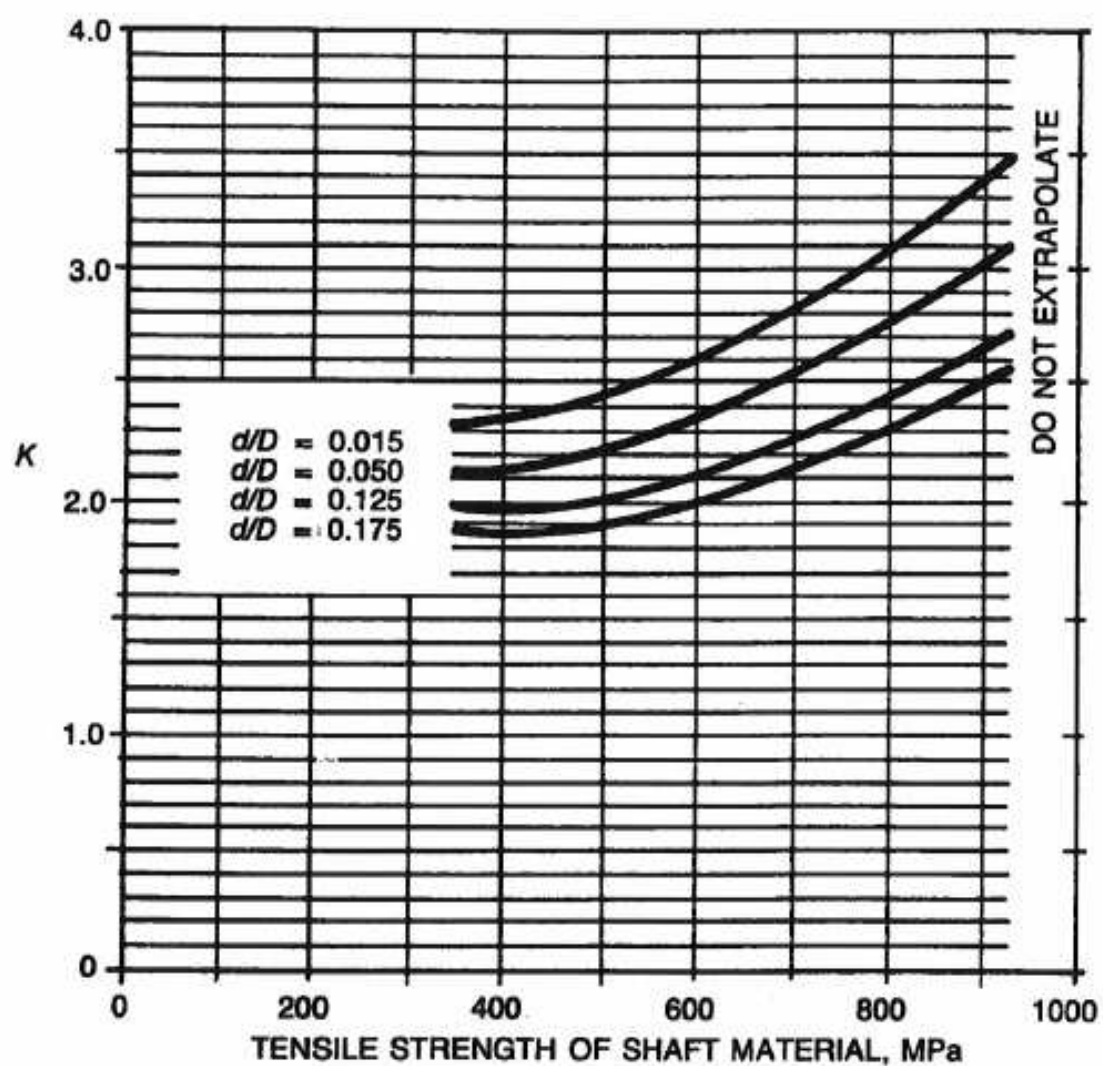
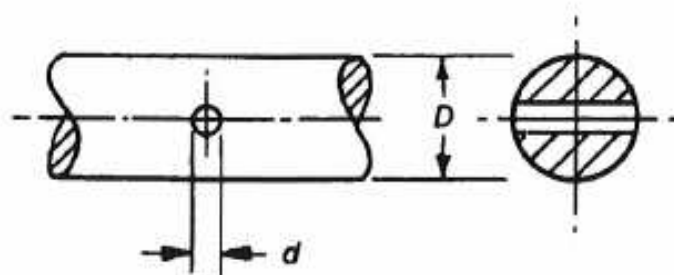


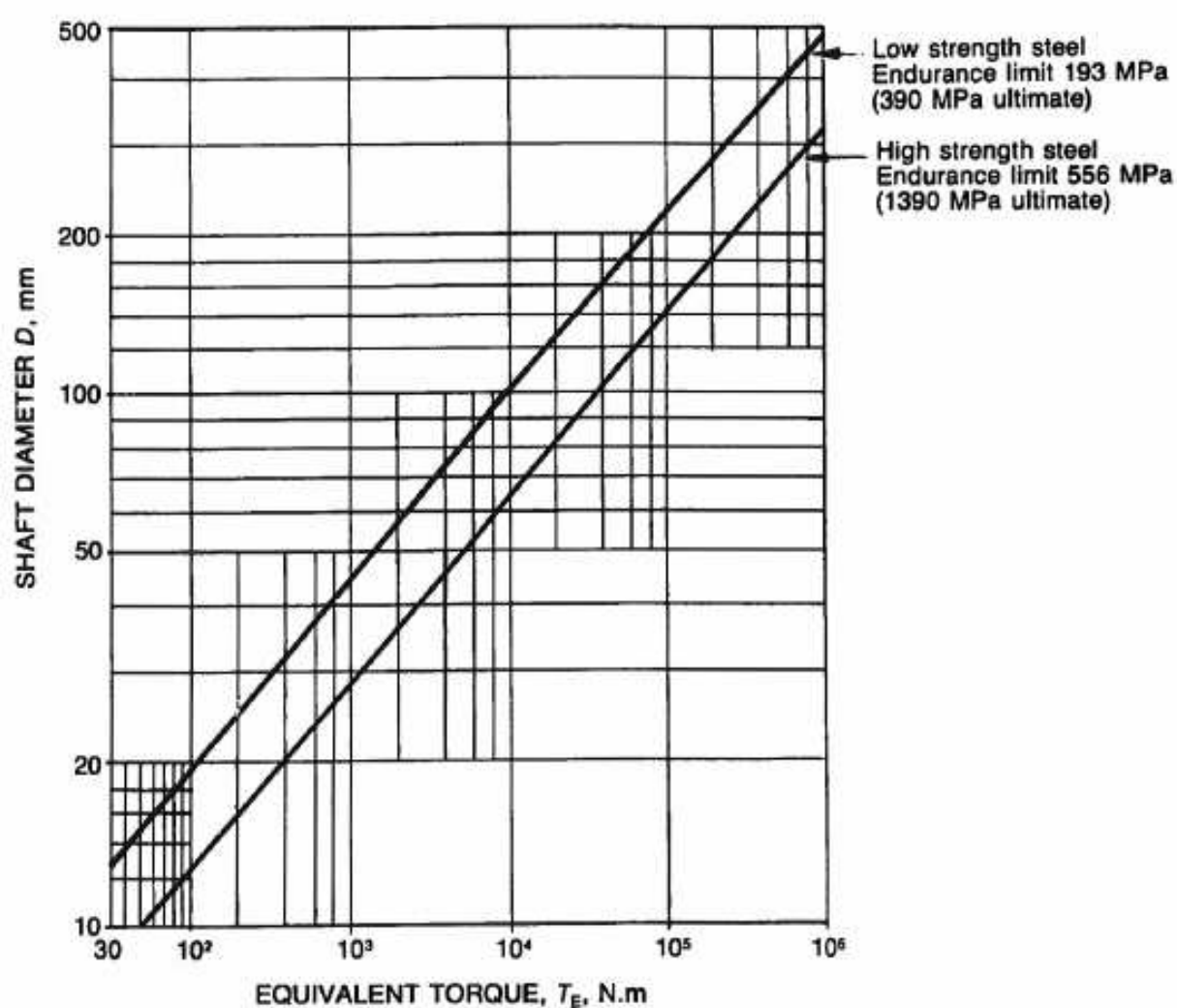
FIGURE 10 STRESS-RAISING FACTOR K FOR SHAFT WITH TRANSVERSE HOLE

APPENDIX A 'TRIAL' SHAFT DIAMETER (Informative)

A 'trial' diameter may need to be assumed in Formulas 1 to 4 given in Table 2.

The 'trial' shaft diameter is to be read directly from Figure A1, which plots shaft diameter against equivalent torque (T_E), where

$$T_E = 1.15 \sqrt{M_q^2 + 0.75T_q^2}$$



NOTE: The curve is based on the following formula:

$$D^3 = \frac{12000 K_s}{F_R} T_E$$

FIGURE A1 'TRIAL' SHAFT DIAMETER

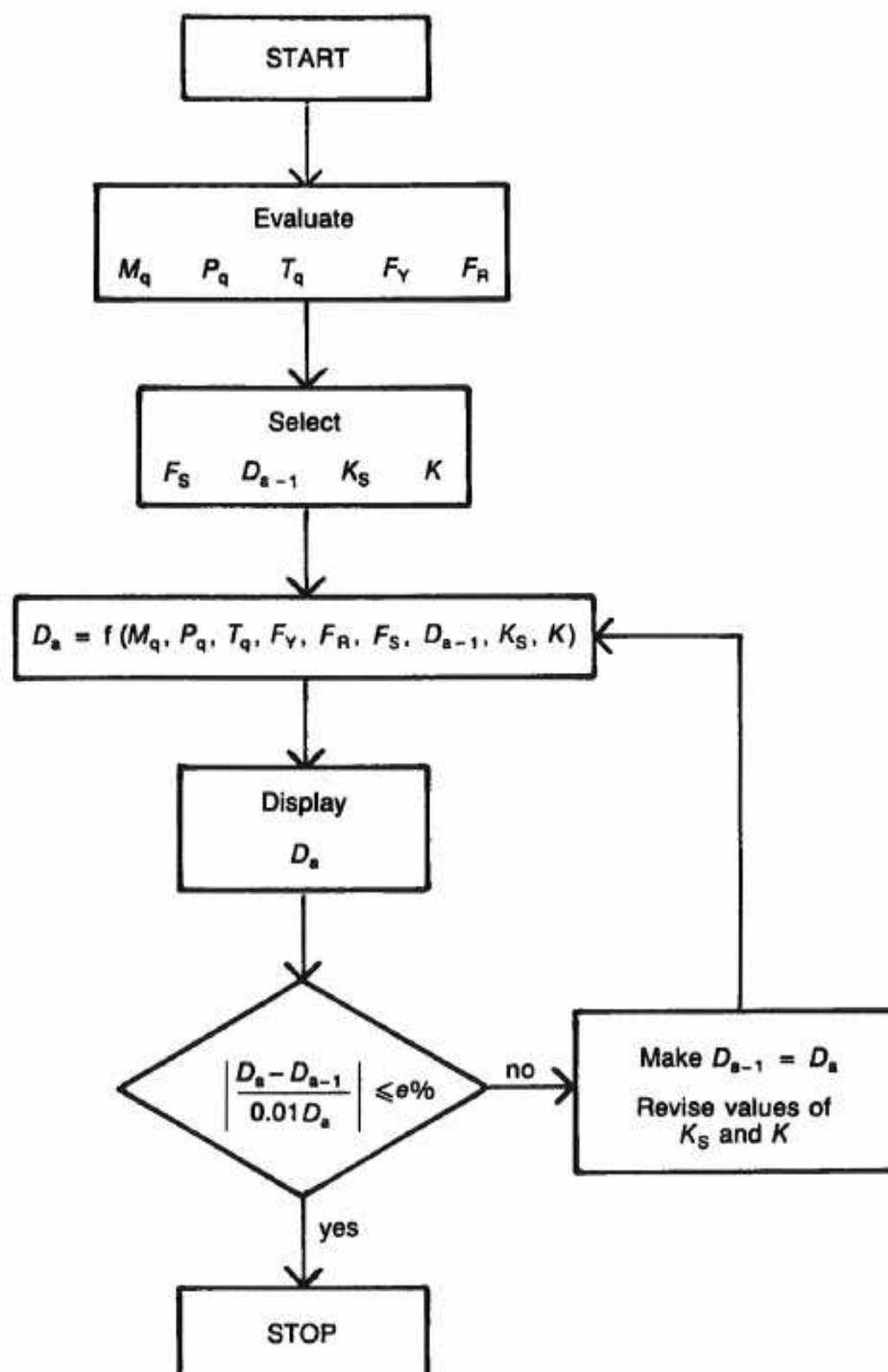
APPENDIX B

ITERATIVE METHOD FOR CALCULATING MINIMUM DIAMETER OF SHAFT

(Informative)

This method is recommended when a programmable calculator is available.

Figure B1 shows the flow chart for a programmable calculator, where one of the Formulas 1 to 4 in Table 2 is used.



NOTES:

- 1 For most applications a value of $e = 1$ would be reasonable, but the designer may feel the need to use a different value.
- 2 Once the final value of D_a has been displayed, this value should be checked by using the appropriate formulas in Table 2.

FIGURE B1 FLOW CHART FOR PROGRAMMABLE CALCULATOR USING
ITERATIVE METHOD

APPENDIX C

CHARACTERISTICS OF MOTOR CONTROLLERS AND TORQUE-LIMITING DEVICES

(Informative)

C1 INTRODUCTION

Motors having high locked-rotor torque (also known as starting torque) or high breakdown torque (also known as pull-out torque) may impose high loads on shafts and other components of the driven mechanism. Care should be taken with the selection of motors and control equipments as the breakdown torque can be as high as 4 times rated torque (also known as full-load torque) for a.c. motors, 3 times for shunt type d.c. motors and 5 times for series type d.c. motors.

The effect of high locked-rotor torque or high breakdown torque can be minimized by electrical means (motor controllers) or mechanical means (fluid coupling or the like).

C2 EFFECT OF MOTOR CONTROLLERS ON SHAFT LOADING

C2.1 Motor controllers for a.c. motors

Motor controllers for a.c. motors can be simple on-off, stepped or automatic 'stemless' control systems.

These motor controller systems result in a torque applied in starting of a value of the order indicated below:

- | | | |
|--|---|---|
| (a) Direct acting with automatic torque regulating system or device, e.g., whole current starter. | — 100% of starting or breakdown torque, whichever is the greater. | |
| (b) Direct acting with automatic torque regulating system or device, e.g., whole current starter with gate type mechanical stepping | — 50% to 80% of starting or breakdown torque whichever is the greater. | |
| (c) Indirect acting with no automatic torque regulating system or device, e.g., pushbutton or master controller | — 100% of starting or breakdown torque, whichever is the greater. | |
| (d) Indirect acting with automatic torque regulating system or device— | | |
| (i) with less than four steps | — 70% | } |
| (ii) with four steps or more, e.g. master controller with time current or voltage control between steps | — 40% | |
| | | of starting or breakdown torque, whichever is the greater |
| (e) Indirect acting with automatic regulated torque system where the accelerating or decelerating motor currents does not vary by more than 20% from the manufacturer's settings, e.g., automatic 'stemless' control | — set up by manufacturer (normally in the vicinity of 150% of rated torque) | |

C2.2 Motor controllers for d.c. motors

The use of whole current and or unregulated controllers for d.c. motors is very limited. Where such devices are used, the maximum possible motor starting torque is to be used in determining the loading of shafts.

These motor controller systems result in a torque applied in starting of a value of the order indicated below:

- (a) Indirect acting controllers with automatic regulating systems or devices of the single or multi-step type, e.g., master controller with suitable time, current or voltage control between steps—
 - (i) shunt or compound motors — 180% of rated torque;
 - (ii) series motors — 250% of rated torque.
- (b) Indirect acting controllers with automatic torque regulating systems where the accelerating and decelerating motor currents does not vary by more than 20% above the manufacturer's setting, e.g., automatic 'stepless' control used with current limit, variable voltage, Ward Leonard or silicon controlled rectifier or equivalent. — set by manufacturer (normally in the vicinity of 150% of rated torque).

C3 EFFECT OF TORQUE-LIMITING DEVICES ON SHAFT LOADING

C3.1 Fluid couplings

Fluid couplings are used to achieve five objectives as follows:

- (a) To reduce the duration of high starting currents.
- (b) To limit the torque applied to the transmission system (usually to twice the maximum rated torque requirement of the driven machine or, where this is not definitely known, to twice the rated torque of the motor).
- (c) To provide controlled acceleration of the driven machine.
- (d) To limit motor rotor inertia in the event of sudden application of overload.
- (e) To absorb load and speed variations.

It is important that the size of fluid couplings be matched to the conditions of each particular application and that the coupling be filled to the correct level to provide the required torque characteristics. An oversized fluid coupling, if filled to an excessive level, will transmit excessive torque and may place an excessive starting load on the motor owing to its own inertia; or if not filled to transmit the correct torque, will have excessive slip under load resulting in reduced efficiency and introducing heat dissipation problems.

As with electric motors, the thermal properties of fluid couplings need to be equated to the duty cycle of the application.

Typical torque-speed curves for a.c. motors and fluid couplings during starting are given in Figure C1.

C3.2 Friction clutches

The torque transmittable by the various types of friction clutch (slipping, centrifugal, pneumatic, magnetic and the like) depends on the characteristics of the particular type (and size) of clutch and the manner in which it is used. Analysis of these factors should indicate a realistic value of applied torque.

C3.3 Eddy current clutches

Eddy current clutches are capable of high degree of torque and speed control. Their torque transmitting properties and method of application may be readily analysed to determine values of applied torque.

NOTE: Different designs of a.c. motors give different torque characteristics, which should be considered when motors are chosen for the various applications.

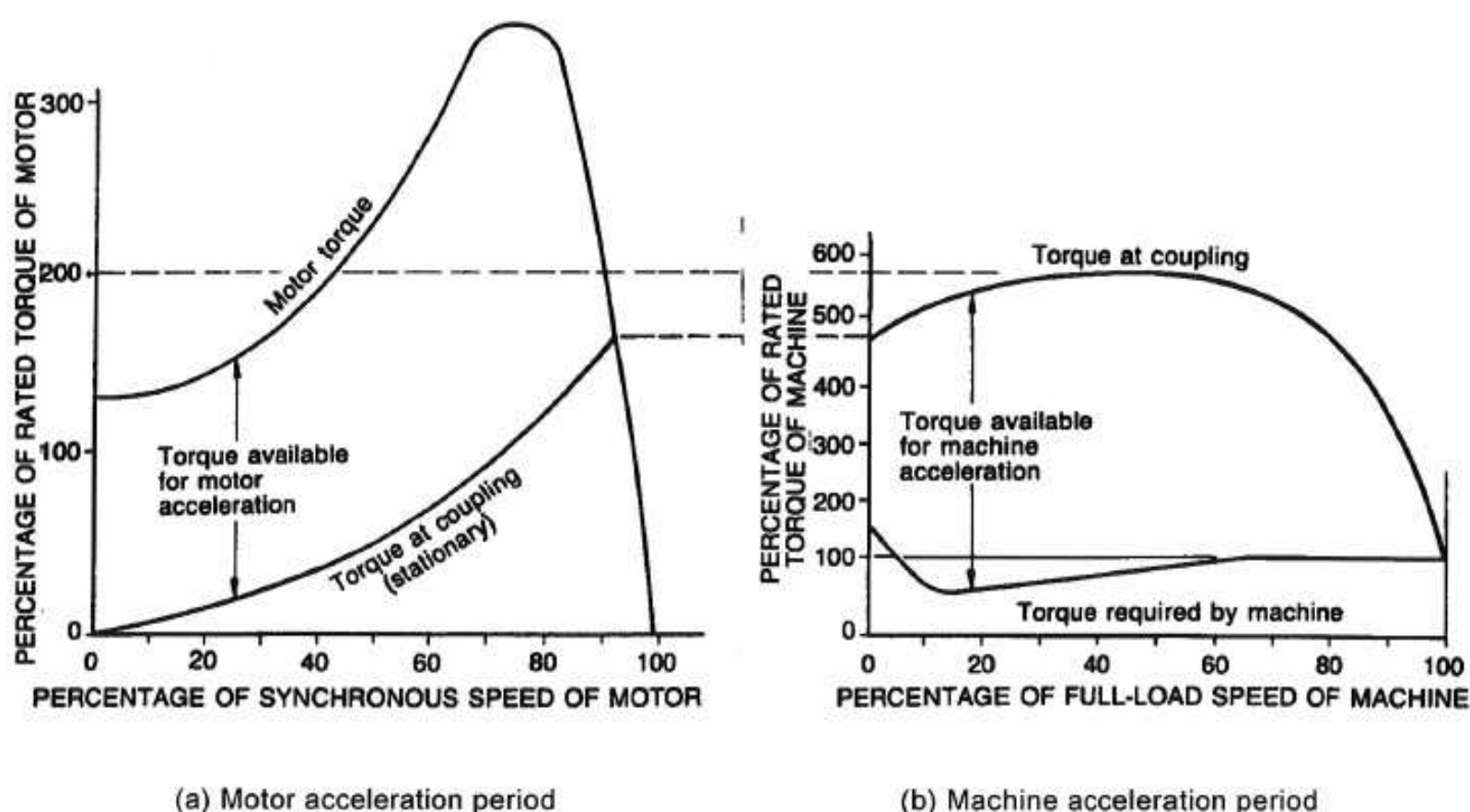


FIGURE C1 TYPICAL TORQUE TRANSMITTED BY FLUID COUPLING DURING STARTING

APPENDIX D

TYPICAL WORKED EXAMPLE—CRANE DRIVING MECHANISM

(Informative)

D1 SCOPE

This Appendix illustrates the application of this Standard in the determination of the maximum torque that an electric motor applies to a crane driving mechanism and in the checking of the diameter of one of the shafts (Shaft 2) in the drive system.

NOTE: Although the inertia effects and the effect of the axial force are quite small they have both been included for completeness of the example. They could safely have been ignored.

D2 DATA**D2.1 General**

An overhead crane is supported on two tracks and is powered by a driven mechanism on each track. The drive mechanism consists of a break-motor driving a trackwheel through a three-stage gear reduction. The crane is supported on each track by two identical trackwheels, one driven, one free-running. Details of the driving mechanism and the crane are shown in Figure D1.

For this example Shaft 2 will be analysed, details of which are shown in Figure D2.

D2.2 Data available to carry out subsequent calculations

(See Figures D1 and D2)

- (a) Motor: Power = 4.3 kW
 Speed = 1360 r/min = 22.67 r/s ($=N_1 = N$)
 Rotational mass moment of inertia (I_M) = 0.011 kg.m²
 Maximum torque = $T_{\max} = 2.6 \times$ rated torque
 Maximum braking torque = 25 N.m
- (b) Gears: All gears are single helical gears with $\beta = 30^\circ$ and have $\psi_n = 20^\circ$ normal pressure angle.
- Pinion 1: Number of teeth $Z_{p1} = 21$
 Normal module $m_n = 4$ mm
 Rotational mass moment of inertia $I_{p1} = 0.003$ kg.m²
- Wheel 1: Number of teeth $Z_{w1} = 66$
 Normal module $m_n = 4$ mm
 Rotational mass moment of inertia $I_{w1} = 0.050$ kg.m²
 PCD = 304.8 mm (PCD = $m_n \times Z / \cos\beta$)
- Pinion 2: Number of teeth $Z_{p2} = 21$
 Normal module $m_n = 5$ mm
 Rotational mass moment of inertia $I_{p2} = 0.004$ kg.m²
 PCD = 121.2 mm (PCD = $m_n \times Z / \cos\beta$)
- Wheel 2: Number of teeth $Z_{w2} = 67$

- Normal module $m_n = 5 \text{ mm}$
 Rotational mass moment of inertia $I_{w2} = 0.157 \text{ kg.m}^2$
- Pinion 3: Number of teeth $Z_{p3} = 23$
 Normal module $m_n = 5 \text{ mm}$
 Rotational mass moment of inertia $I_{p3} = 0.015 \text{ kg.m}^2$
- Wheel 3: Number of teeth $Z_{w3} = 79$
 Normal module $m_n = 5 \text{ mm}$
 Rotational mass moment of inertia $I_{w3} = 0.305 \text{ kg.m}^2$
- Trackwheel: Diameter $D_{TW} = 500 \text{ mm}$
 $I_{TW} = 6.32 \text{ kg.m}^2$
- The efficiency of power transmission for each gear pair $(\eta_{1/2}, \eta_{2/3}, \eta_{3/4}) = 96\%$
- (c) Crane details: Safe working load 25 000 kg
 Mass of crane 43 000 kg
 Crane supported by two drive wheels and two idler wheels; all wheels equally loaded.
- (d) Details of shaft 2: Material: Plain carbon steel
 Tensile strength $F_U = 500 \text{ MPa}$
 Endurance limit $F_R = 0.45 F_U = 225 \text{ MPa}$
 Yield strength $F_Y = 350 \text{ MPa}$

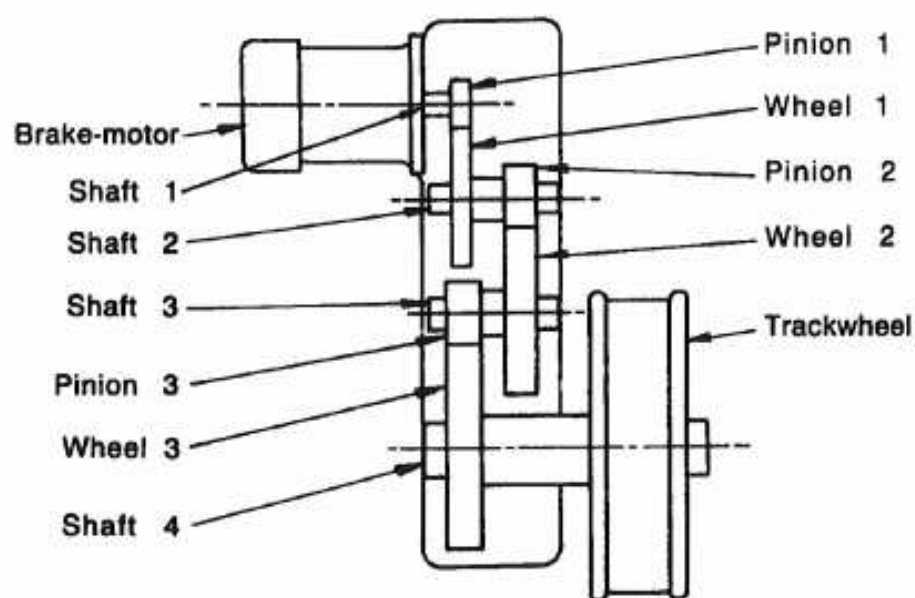


FIGURE D1 DIAGRAM OF DRIVE SYSTEM

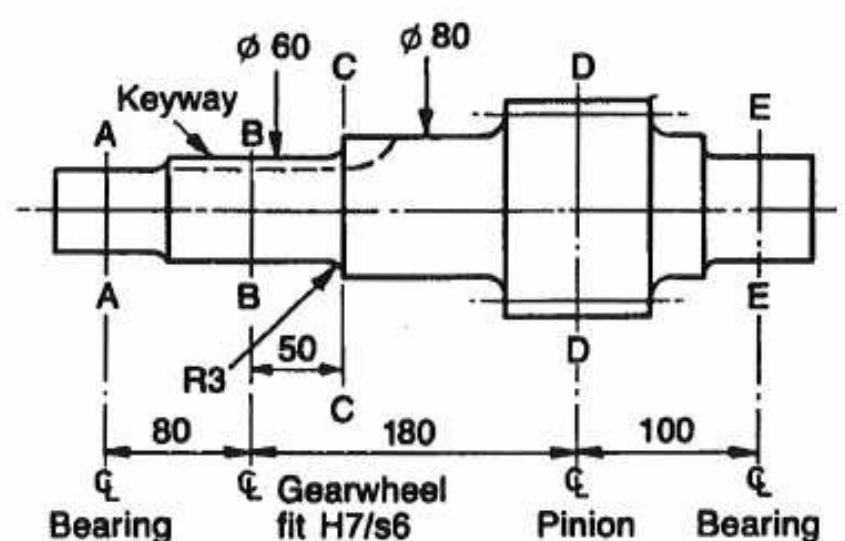


FIGURE D2 DETAIL OF SHAFT 2

D3 CALCULATIONS.

D3.1 General

The helical gears will produce both axial forces and bending moments, as well as a transmitted torque. The maximum values of axial force, bending moment and torque at the most critical cross-section are to be used in the analysis.

D3.2 Selection of the appropriate formula

A crane of the nature described in Paragraph D2.1 would be expected to have more than 600 starts per year and the number of revolutions per year of the shaft to be much greater than 900. The crane is subject to braking as well as driving, so the torque on the shaft is reversing; therefore, Formula 3 of Table 2 is applicable.

D3.3 Basic calculations

D3.3.1 Calculation of rotational speeds

$$N_1 = N = 22.67 \text{ r/s}$$

$$N_2 = N_1 \frac{Z_{p1}}{Z_{w1}} = 22.67 \frac{21}{66} = 7.21 \text{ r/s}$$

$$N_3 = N_2 \frac{Z_{p2}}{Z_{w2}} = 7.21 \frac{21}{67} = 2.26 \text{ r/s}$$

$$N_4 = N_3 \frac{Z_{p3}}{Z_{w3}} = 2.26 \frac{23}{79} = 0.658 \text{ r/s}$$

D3.3.2 Calculation of moments of inertia

Moment of inertia of shaft 1

$$I_1 = I_{R1} = I_M + I_{P1} = 0.011 + 0.003 = 0.014 \text{ kgm}^2 (= I_{E1})$$

Moment of inertia of shaft 2

$$I_2 = I_{W1} + I_{P2} = 0.050 + 0.004 = 0.054 \text{ kgm}^2$$

$$I_{R2} = I_2 \left(\frac{N_2}{N} \right)^2 = 0.054 \left(\frac{7.21}{22.67} \right)^2 = 0.0055 \text{ kgm}^2 (= I_{E2})$$

Moment of inertia of shaft 3

$$I_3 = I_{W2} + I_{P3} = 0.157 + 0.015 = 0.172 \text{ kgm}^2$$

$$I_{R3} = I_3 \left(\frac{N_3}{N} \right)^2 = 0.172 \left(\frac{2.26}{22.67} \right)^2 = 0.00171 \text{ kgm}^2 (=I_{E3})$$

Moment of inertia of shaft 4

$$I_4 = I_{W3} + 2 I_{TW} = 0.305 + 2 \times 6.32 = 12.945 \text{ kgm}^2$$

$$I_{R4} = I_4 \left(\frac{N_4}{N} \right)^2 = 12.945 \left(\frac{0.658}{22.67} \right)^2 = 0.0109 \text{ kgm}^2$$

For further calculations the mass driven by the motor (m_1) and the driving speed (V_1) is determined.

As there are two motors on the crane, one for each track (see Paragraph D2.1),

$$m_1 = \frac{\text{Safe working load} + \text{Mass of crane}}{2} = \frac{25\,000 + 43\,000}{2} = 34\,000 \text{ kg}$$

The speed of the crane is calculated using the radius and the rotational speed of the trackwheel, that is—

$$V_1 = 2\pi N_4 \times \frac{D_{TW}}{2} = 2\pi \times 0.658 \times \frac{0.500}{2} = 1.034 \text{ m/s}$$

The equivalent rotational mass moment of inertia of the crane and the load is—

$$I_{L1} = m_1 \left(\frac{V_1}{2\pi N} \right)^2 = 34\,000 \left(\frac{1.034}{2\pi \times 22.67} \right)^2 = 1.792 \text{ kgm}^2$$

$$\text{Thus } I_{E4} = I_{R4} + I_{L1} = 0.0109 + 1.792 = 1.803 \text{ kgm}^2$$

D3.4 Analysis of the driving system

D3.4.1 Basic torque calculations

$$\text{Rated motor power} = \frac{2\pi N \times \text{rated torque}}{1000} \text{ (kW)}$$

From this equation—

$$\text{Rated torque} = \frac{1000 \times \text{rated motor power}}{2\pi N} = \frac{1000 \times 4.3}{2\pi \times 22.67} = 30.2 \text{ N.m}$$

Accordingly, the starting torque ($T_{\max.}$) will be (see Paragraph D2.2(a))—

$$T_{\max.} = 2.6 \times \text{rated torque} = 78.5 \text{ N.m}$$

NOTE: As the maximum brake torque of 25 N.m (see Paragraph D2.2(a)) is less than the starting torque, $T_{\max.}$ is the torque to be used for design purposes.

D3.4.2 Calculation of the angular acceleration of the driving medium

NOTE: The purpose of this calculation is to find the input torque in shaft 2. The formula from Table 1 shows that the angular acceleration of the motor is required. This acceleration is obtained by equating the output torque of shaft 4 and the torque required to drive the crane.

The formula for the output torque of shaft 4 in Table 1 is—

$$T_{4 \text{ out}} = \left\{ \left[(T_m - I_{E1}\alpha) \frac{N_1}{N_2} \eta_{1/2} - I_{E2} \frac{N_1}{N_2} \alpha \right] \frac{N_2}{N_3} \eta_{2/3} - I_{E3} \frac{N_1}{N_3} \alpha \right\} \frac{N_3}{N_4} \eta_{3/4} - I_{E4} \frac{N_1}{N_4} \alpha$$

The load torque at the trackwheel shaft is due mainly to the rolling resistance of the trackwheels. The load on each pair of trackwheels is m_1g (see Figure D3).

In Figure D3 γ = maximum friction raising factor for flanged wheels with rolling element bearings = 2.0

$\ast f$ = maximum eccentricity of the contact point = 0.0005 m

$\ast d$ = pitch circle diameter of rolling element bearing = 0.15 m

$\ast \mu$ = maximum coefficient of friction of rolling element bearings = 0.002

Taking all moments about centre O, we find the resisting torque at the trackwheel shaft. As this has to be overcome by $T_{4 \text{ out}}$, we find—

$$\begin{aligned} T_{4 \text{ out}} &= \left(\mu m_1 g \frac{d}{2} + m_1 g f \right) \gamma = m_1 g \left(\frac{\mu d}{2} + f \right) \gamma \\ &= 34\,000 \times 9.81 \left(\frac{0.002 \times 0.15}{2} + 0.0005 \right) 2.0 \\ &= 433.6 \text{ N.m} \end{aligned}$$

By substituting the value of $T_{4 \text{ out}}$ in the basic formula (above) from Table 1 the following equation is obtained:

$$433.6 = \left\{ \left[(78.5 - 0.014\alpha) \frac{22.67}{7.21} 0.96 - 0.0055 \frac{22.67}{7.21} \alpha \right] \frac{7.21}{2.26} 0.96 - 0.00171 \frac{22.67}{2.26} \alpha \right\} \times \frac{2.26}{0.658} 0.96 - 1.803 \frac{22.67}{0.658} \alpha$$

Thus $\alpha = 31.21 \text{ rad/s}^2$

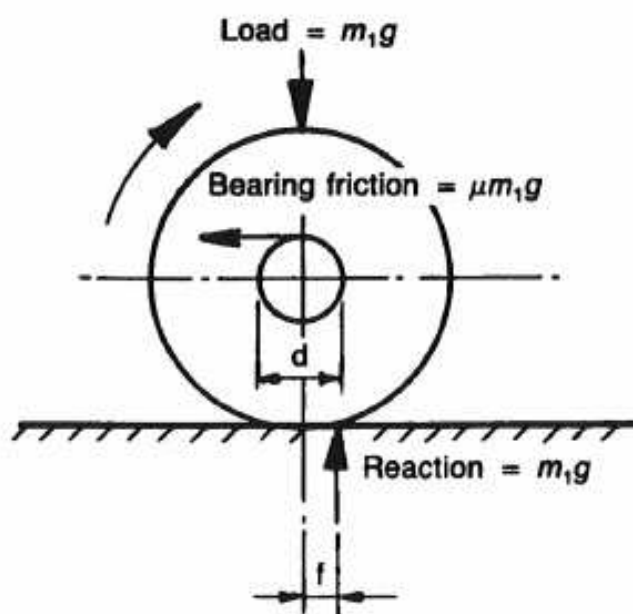


FIGURE D3 DIAGRAM OF FORCES ACTING ON TRACKWHEEL

* Data obtained from literature, textbooks and manufacturers' catalogues.

D3.4.3 Calculation of input torque for shaft 2

From Table 1,

$$T_2 = (T_M - I_{E1} \alpha) \frac{N_1}{N_2} \eta_{1/2} = (78.5 - 0.014 \times 31.21) \frac{22.67}{7.21} \times 0.96 = 235.6 \text{ N.m } (= T_q)$$

D3.4.4 Calculation of forces arising at the gears and reactions arising at the bearings for shaft 2

NOTE: These calculations are necessary for the eventual calculations of the bending moments.

$$\text{Tangential force at gear } F_t = \frac{\text{torque being transmitted}}{\frac{\text{PCD}}{2}}$$

$$\text{for Wheel 1 } F_{tw1} = \frac{235.6}{0.1524} = 1546 \text{ N}$$

$$\text{for Pinion 2 } F_{tp2} = \frac{235.6}{0.0606} = 3888 \text{ N}$$

$$\text{Separating forces at gear } F_s = F_t \times \frac{\tan \psi_n}{\cos \beta} = F_t \times \frac{\tan 20^\circ}{\cos 30^\circ}$$

$$\text{for wheel 1 } F_{rw1} = 1546 \times \frac{\tan 20^\circ}{\cos 30^\circ} = 650 \text{ N}$$

$$\text{for Pinion 2 } F_{rp2} = 3888 \times \frac{\tan 20^\circ}{\cos 30^\circ} = 1634 \text{ N}$$

$$\text{Axial forces at gear } F_a = F_t \times \tan \beta = F_t \times \tan 30^\circ$$

$$\text{for wheel 1 } F_{aw1} = 1546 \times \tan 30^\circ = 893 \text{ N}$$

$$\text{for Pinion 2 } F_{ap2} = 3888 \times \tan 30^\circ = 2245 \text{ N}$$

The gear forces acting on shaft 2 are shown in Figure D4.

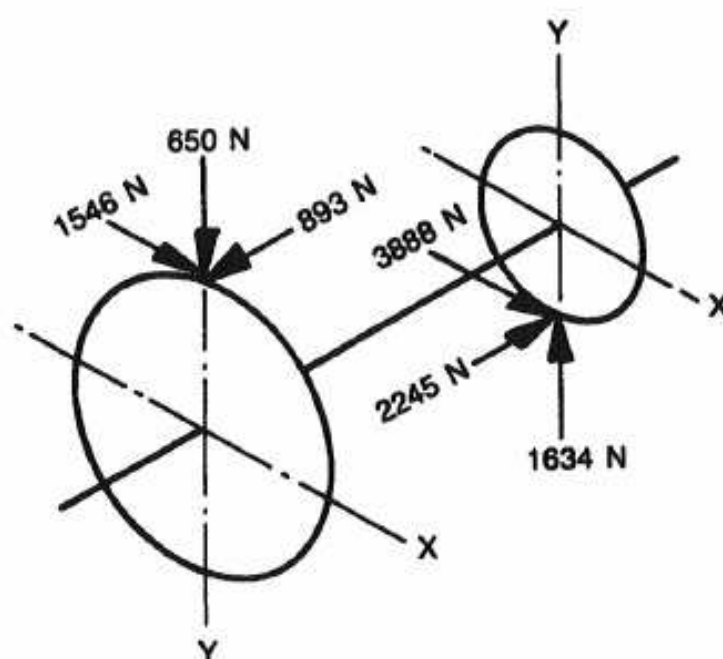


FIGURE D4 GEAR FORCES ACTING ON SHAFT 2

The bending moment diagrams for the Z-X (horizontal) and Z-Y (vertical) planes can now be drawn and the critical cross-sections selected (see Figures D5 and D6).

Calculation of reaction forces in Z-X plane—

$$R_{AX} = \frac{(1546 \times 280) + (3888 \times 100)}{360} = 2882 \text{ N}$$

$$R_{EX} = (1546 + 3888) - 2282 = 3152 \text{ N}$$

Calculation of reaction forces in Z-Y plane—

$$R_{AY} = \frac{(650 \times 280) + (893 \times 152.4) + (2245 \times 60.6) - (1643 \times 100)}{360} = 808 \text{ N}$$

$$R_{EY} = (1634 - 650) + 808 = 1792 \text{ N}$$

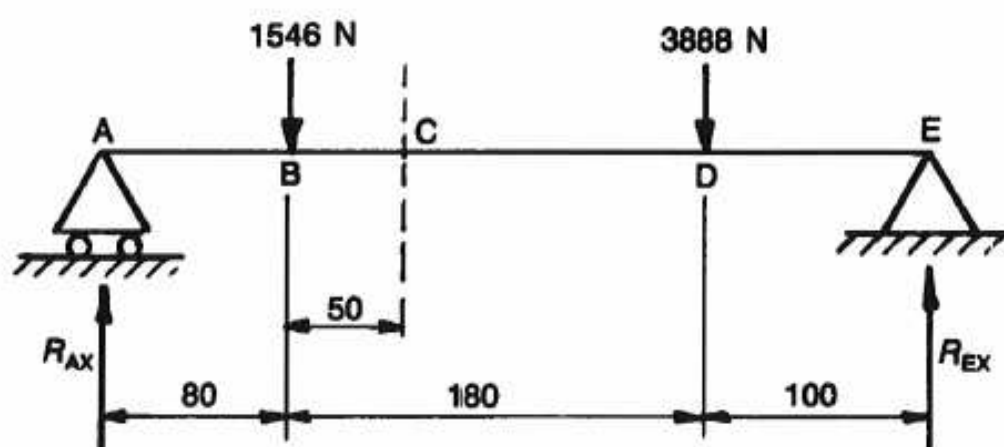


FIGURE D5 FORCES ACTING ON SHAFT 2 IN Z-X (HORIZONTAL) PLANE

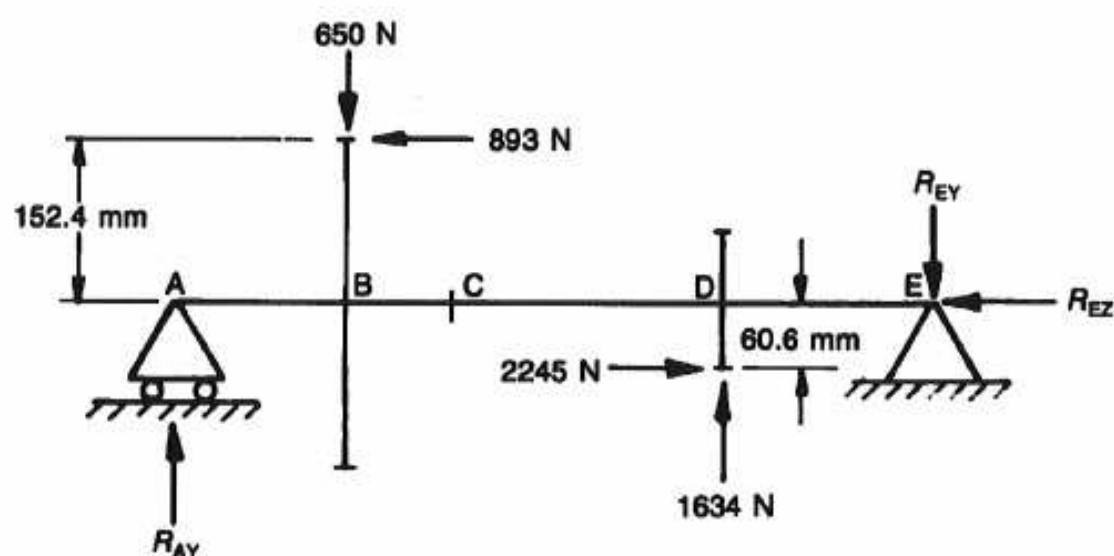


FIGURE D6 FORCES ACTING ON SHAFT 2 IN Z-Y (VERTICAL) PLANE

D3.5 Calculations of bending moments

To assess the critical section for shaft 2, bending moment diagrams in both horizontal and vertical planes are drawn up (see Figures D7 and D8).

NOTES:

- 1 It can be determined from Figures D7 and D8 that the maximum bending moment on the 60 mm diameter part of the shaft occurs at section 'C'. The maximum bending moment on the 80 mm diameter part of the shaft occurs at section 'D'.
- 2 A quick calculation will show that even though the bending moment at section 'D' is greater than that at section 'C', the shaft will be more highly stressed at the smaller diameter i.e. at section 'C'.

Bending moments at section 'C'—

In Z-X plane, $M_{zx} = (2282 \times 0.13) - (1546 \times 0.05) = 219.4 \text{ N.m}$

In Z-Y plane, $M_{zy} = (808 \times 0.13) - (650 \times 0.05) - (893 \times 0.1524) = -63.6 \text{ N.m}$

The resultant bending moment is—

$$M_C = M_2 = \sqrt{(M_{zx}^2 + M_{zy}^2)} = 228.4 \text{ N.m} (= M_q)$$

Axial force at section 'C' (tension or compression depending on direction of motion)

$$P_2 = 893 \text{ N} (= P_q)$$

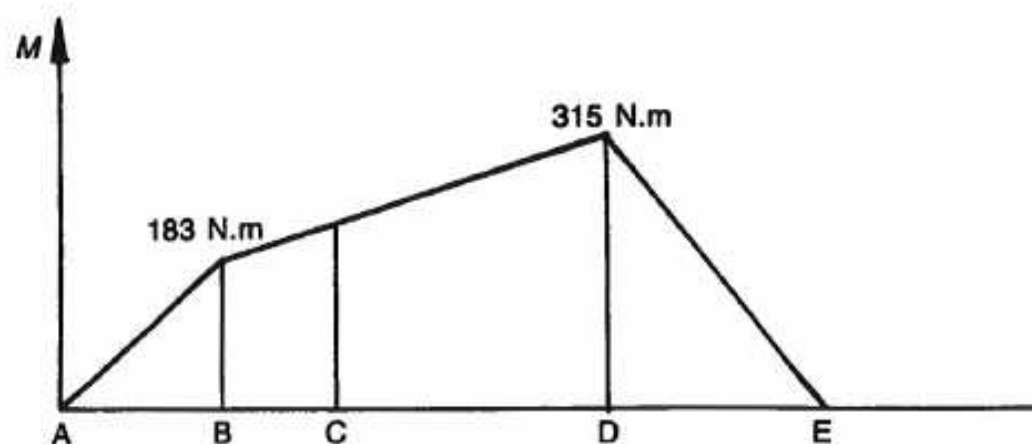


FIGURE D7 BENDING MOMENT DIAGRAM IN Z-X (HORIZONTAL) PLANE

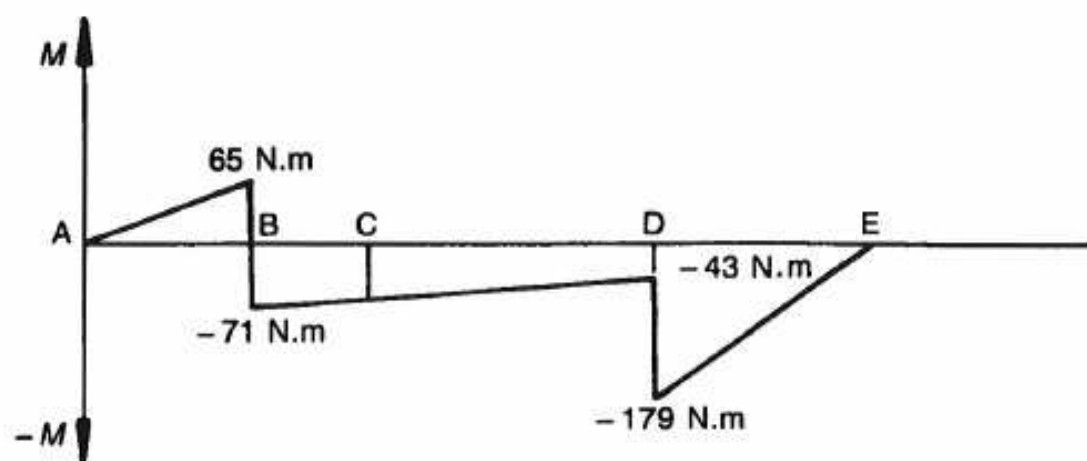


FIGURE D8 BENDING MOMENT DIAGRAM IN Z-Y (VERTICAL) PLANE

D3.6 Calculation of shaft diameter

D3.6.1 Selection of various factors

- (a) *Size factor (K_s)* K_s is selected from Figure 1 and as section 'C' is 60 mm dia. it is taken as $K_s = 1.52$
- (b) *Geometry factor (K)* As the shaft (see Figure D2) has both a step and a keyway as stress-raisers, according to Clause 8.2(d) both effects have to be considered together.
- (i) Correction factor Δ is selected from Figure 3

$$\frac{D_1}{D} = \frac{80}{60} = 1.33, \text{ therefore } \Delta = 0.05$$

- (ii) Calculation of Z (see Figure 4)

$$Z = \frac{R}{D} + \Delta = \frac{3}{60} + 0.05 = 0.10$$

- (iii) Selection of K from Figure 4

$$K = 1.55 \text{ (for step)}$$

- (iv) Selection of K from Figure 7

$$K = 2.22 \text{ (for keyway)}$$

According to Clause 8.2(d),

$$K = K_{\text{keyway}} + 0.2 \times K_{\text{step}} = 2.22 + 0.2 \times 1.55 = 2.53$$

D3.6.2 Calculation of minimum shaft diameter

NOTE: This calculation is made for section 'C', i.e., where the actual diameter (smaller) is 60 mm.

For this example, Formula 3 of Table 2 is applicable, that is.—

$$D^3 = \frac{10^4 F_s}{F_R} K_s K \sqrt{\left(M_q \frac{P_q D}{8000} \right)^2 + \frac{3}{4} T_q^2}$$

$$D^3 = \frac{10^4 \times 1.2}{225} 1.52 \times 2.53 \sqrt{\left(228.4 \frac{893 \times 60}{8000} \right)^2 + 0.75 \times 235.6^2}$$

from which $D = 40.0$ mm

D3.6.3 Checking of actual shaft diameter

The actual diameter of the shaft at section 'C' is 60 mm, which is well above the calculated minimum diameter of 40.0 mm and thus satisfies Clause 6.2(a).

D3.7 'Trial' diameter method

Another way of calculating the minimum diameter of the shaft is by the method described in Appendix A.

A 'trial' diameter, resulting from this method, is substituted into the relevant minimum diameter formula, in this example Formula 3 of Table 2.

According to the formula in Paragraph A1—

$$T_E = 1.15 \sqrt{M_q^2 + 0.75 T_q^2} = 1.15 \sqrt{228.4^2 + 0.75 \times 235.6^2} = 352.2 \text{ N.m}$$

As the material used for the shaft is of low tensile strength ($F_U = 500$ MPa, see Paragraph D2.2(d)), using Figure A1 we read off as trial diameter $D \approx 31$ mm.

Once again, the factors are selected, now for $D = 31$ mm

From Table 2, $F_s = 1.2$

From Figure 1, $K_s = 1.29$

From Figure 3, $\Delta = 0$ as $\frac{D_1}{D} = \frac{80}{31} = 2.58$

From Figure 4, $Z = \frac{3}{31} + 0 = 0.097$ and thus $K_{\text{step}} = 1.54$

From Figure 7, $K_{\text{keyway}} = 2.22$

and according to Clause 8.2(d)

$$K = K_{\text{keyway}} + 0.2 \times K_{\text{step}} = 2.22 + 0.2 \times 1.54 \approx 2.53$$

Substituting all known figures into Formula 3 of Table 2 we obtain

$$D^3 = \frac{10^4 \times 1.2}{225} 1.29 \times 2.53 \sqrt{\left(228.4 \frac{893 \times 31}{8000}\right)^2 + 0.75 \times 235.6^2}$$

$$D = 37.7 \text{ mm}$$

The actual diameter of 60 mm is well above the minimum of 37.7 mm calculated with this method.

APPENDIX E TYPICAL WORKED EXAMPLE—CRANE HOIST DRIVE (Informative)

E1 SCOPE

This Appendix illustrates the application of this Standard to a crane hoist drive shown schematically in Figure E1.

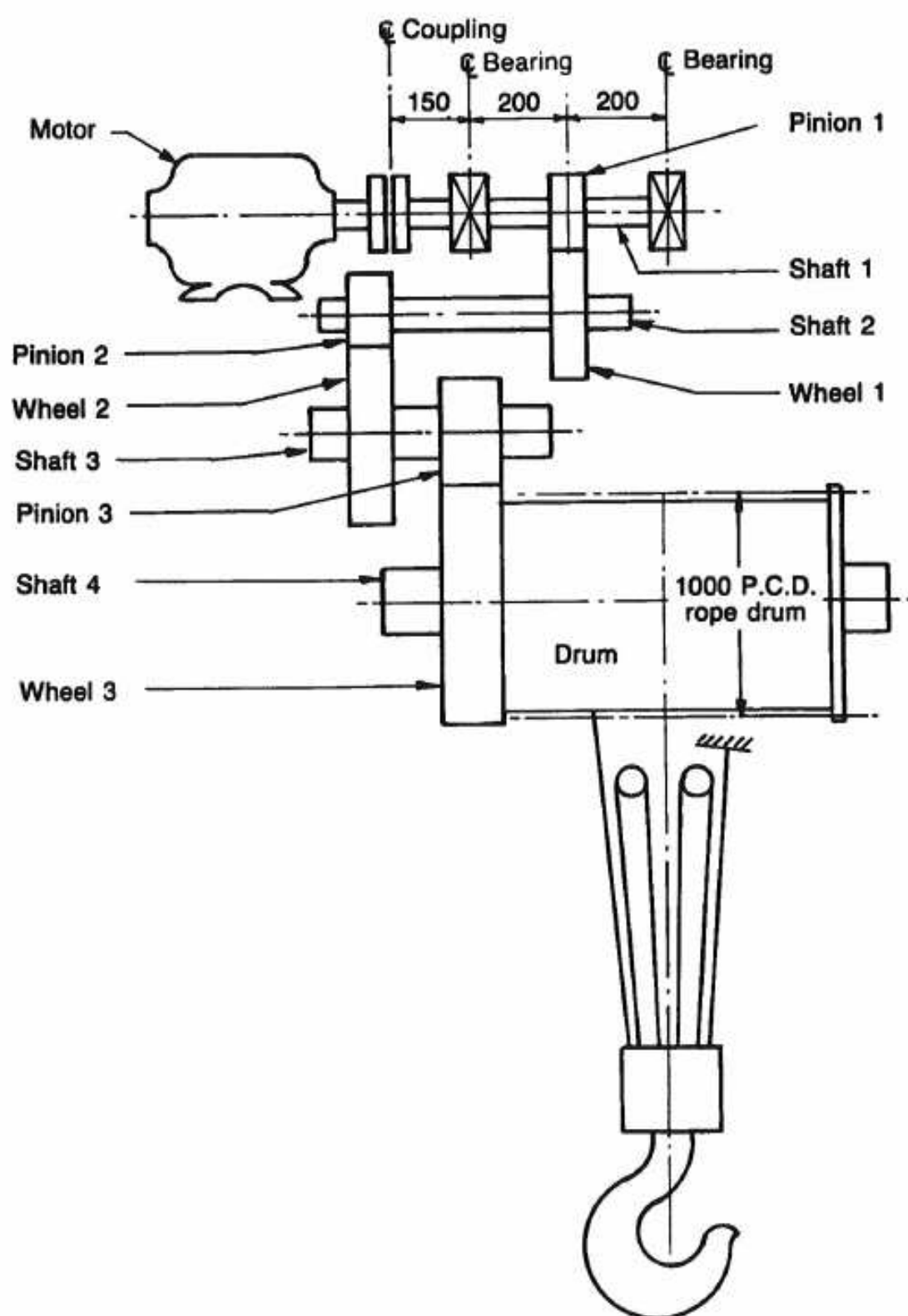


FIGURE E1 SCHEMATIC DRAWING OF CRANE HOIST DRIVE

E2 DATA

E2.1 General

The load to be lifted vertically is 50 t. The mass of the hook block is 300 kg. The centre-line of the gear train is horizontal. The six falls of rope are arranged such that one end of the rope is anchored to the drum, the other to the trolley.

The first motion shaft is keyed to accept both the pinion (component fit H7/s6) and coupling. Two identical spherical roller bearings are used to locate the first motion shaft. The helical gear (pinion 1) and the fixed bearing are arranged such that the axial force resulting from the mesh of the gears subjects the shaft to a compressive force when the hoist is lifting the load. In this example, changing the direction of rotation does not reverse the axial load in the shaft.

The diameter of the first motion shaft needs to be calculated at the centre-line of pinion 1, knowing that the shaft material has a tensile strength (F_u) of 500 MPa.

E2.2 Data available to carry out subsequent calculations.

- (a) Motor: Power = 100 kW
 Speed = 1440 r/min = 24 r/s ($= N_1 = N$)
 Rotational mass moment of inertia (I_M) = 1.274 kg.m²

$$\frac{\text{Breakdown (pullout) torque}}{\text{Rated (full load) torque}} = 3.75 \frac{\text{Locker - roter (starting) torque}}{\text{Rated (full load) torque}} = 1.4$$

Automatic torque regulation of the motor is achieved by voltage control over four steps.

The motor incorporates a brake, adjusted to a maximum braking torque of 1000 N.m.

- (b) Coupling: Rotational mass moment of inertia (I_c) = 0.017 kg.m²
- (c) Gears: All gears are single helical gears with $\beta = 30$ degrees and have $\psi_n = 20$ degrees normal pressure angle.
- Pinion 1: Number of teeth $Z_{p1} = 21$
 Normal module $m_n = 5$ mm
 Rotational mass moment of inertia $I_{p1} = 0.007$ kg.m²
- Wheel 1: Number of teeth $Z_{w1} = 63$
 Normal module $m_n = 5$ mm
 Rotational mass moment of inertia $I_{w1} = 0.540$ kg.m²
- Pinion 2: Number of teeth $Z_{p2} = 18$
 Normal module $m_n = 7$ mm
 Rotational mass moment of inertia $I_{p2} = 0.020$ kg.m²
- Wheel 2: Number of teeth $Z_{w2} = 81$
 Normal module $m_n = 7$ mm
 Rotational mass moment of inertia $I_{w2} = 8.500$ kg.m²
- Pinion 3: Number of teeth $Z_{p3} = 24$
 Normal module $m_n = 9$ mm
 Rotational mass moment of inertia $I_{p3} = 0.240$ kg.m²

Wheel 3 and Drum:	Number of teeth $Z_{w3} = 134$ Normal module $m_n = 9$ mm Rotational mass moment of inertia $I_4 = 2500$ kg.m ²
Efficiency:	The efficiency of power transmission for each gear pair ($\eta_{1/2}$, $\eta_{2/3}$, $\eta_{3/4}$) = 96 percent

E3 ANALYSES OF LOADS FOR THE PURPOSE OF DETERMINING THE MINIMUM DIAMETER OF SHAFT 1

E3.1 Basic calculations

E3.1.1 Calculation of rotational speeds

$$N_1 = N = 24 \text{ r/s}$$

$$N_2 = N_1 \frac{Z_{p1}}{Z_{w1}} = 24 \times \frac{21}{63} = 8 \text{ r/s}$$

$$N_3 = N_2 \frac{Z_{p2}}{Z_{w2}} = 8 \times \frac{18}{81} = 1.78 \text{ r/s}$$

$$N_4 = N_3 \frac{Z_{p3}}{Z_{w3}} = 1.78 \times \frac{24}{134} = 0.32 \text{ r/s}$$

E3.1.2 Calculation of moment of inertia.

Moment of inertia of shaft 1.

$$I_1 = I_{R1} = I_M + I_C + I_{P1} = 1.274 + 0.017 + 0.007 = 1.298 \text{ kg.m}^2 (= I_{E1})$$

Moment of inertia of shaft 2.

$$I_2 = I_{w1} + I_{p2} = 0.540 + 0.020 = 0.560 \text{ kg.m}^2$$

$$I_{R2} = I_2 \left(\frac{N_2}{N} \right)^2 = 0.560 \times \left(\frac{8}{24} \right)^2 = 0.062 \text{ kg.m}^2 (= I_{E1})$$

Moment of inertia of shaft 3.

$$I_3 = I_{w2} + I_{p3} = 8.500 + 0.240 = 8.740 \text{ kg.m}^2$$

$$I_{R3} = I_3 \left(\frac{N_3}{N} \right)^2 = 8.740 \times \left(\frac{1.78}{24} \right)^2 = 0.048 \text{ kg.m}^2 (= I_{E1})$$

Moment of inertia of shaft 4.

$$I_4 = 2500 \text{ kg.m}^2$$

$$I_{R4} = I_4 \left(\frac{N_4}{N} \right)^2 = 2500 \times \left(\frac{0.32}{24} \right)^2 = 0.0444 \text{ kg.m}^2$$

The equivalent rotational mass moment of inertia of the load and hook is

$$I_{L1} = m_1 \left(\frac{V_1}{2\pi N} \right)^2 = (50\,000 + 300) \times \left(\frac{0.168}{2\pi \times 24} \right)^2 = 0.062 \text{ kg.m}^2$$

NOTE:

$$V_1 = \frac{\frac{D_{\text{DRUM}}}{2} \times 2\pi N_4}{f} = \frac{0.5 \times 2\pi \times 0.32}{6} = 0.168 \text{ m/s}$$

where f = number of falls of rope

$$\text{Thus } I_{E4} = I_{R4} + I_{L1} = 0.444 + 0.062 = 0.506 \text{ kg.m}^2$$

E3.2 Analysis while hoisting the maximum load

E3.2.1 Basic torque calculations

$$\text{Full load torque} = \frac{\text{Rated power in kilowatts} \times 1000}{2\pi \times N} = \frac{100 \times 1000}{2\pi \times 24} = 663.1 \text{ N.m}$$

$$\text{Breakdown (pullout) torque} = 3.75 \times 663.1 = 2487 \text{ N.m (see Paragraph E2.2(a)).}$$

$$\text{Torque of driving motor} = I_M = 0.4 \times \text{Breakdown (pullout) torque} = 0.4 \times 2487 = 995 \text{ N.m (see Paragraph C2.1(d)(ii)).}$$

E3.2.2 Calculation of the angular acceleration of the driving medium.

NOTE: The purpose of this calculation is to find the output torque in Shaft 1. The formula from Table 1 shows that the angular acceleration of the motor is required. This acceleration is obtained by equating the output torque of shaft 4 and the torque required to lift the load.

The formula for the output torque of shaft 4 in Table 1 may be simplified to—

$$T_{4 \text{ out}} = \left[(T_M - I_{E1}\alpha)\eta_{1/2}\eta_{2/3}\eta_{3/4} - I_{E2}\alpha\eta_{2/3}\eta_{3/4} - I_{E3}\alpha\eta_{3/4} - I_{E4}\alpha \right] \frac{N}{N_4}$$

$$\text{Because of the reeving arrangement, the static force in the rope attached to the drum is } \frac{50\,300 \times 9.81}{6} \text{ N}$$

Thus, the torque required to overcome the load resistance is—

$$T_{\text{DRUM}} = \frac{50\,300 \times 9.81}{6} \times 0.5 = 41\,120 \text{ N.m}$$

By equating the required torque and the output torque on shaft 4, i.e. $T_{4 \text{ out}} = T_{\text{DRUM}}$, we obtain the following equation:

$$\left[(995 - 1.298\alpha)0.96^3 - 0.062\alpha \times 0.96^2 - 0.048\alpha \times 0.96 - 0.506\alpha \right] \frac{24}{0.32} = 41\,120$$

$$\text{i.e., } 880.3 - 1.148\alpha - 0.057\alpha - 0.046\alpha - 0.506\alpha = 548.3$$

$$\text{From which we obtain } \alpha = \frac{332.0}{1.757} = 189.0 \text{ rad/s}^2$$

E3.2.3 Calculation of the output torque for shaft 1

$$T_{1 \text{ out}} = T_M - I_{E1}\alpha = 995 - 1.298 \times 189.0 = 749.7 \text{ N.m (= } T_q)$$

E3.2.4 Calculation of forces arising at the gear and reactions arising at the bearings for shaft 1 (see Figure E2)

NOTE: These calculations are necessary for the eventual calculations of the bending moments.

$$\text{Tangential force at gear } F_t = \frac{\text{Torque being transmitted}}{\frac{\text{PCD}}{2}}$$

As the pitch circle diameter $PCD = Z \times m_t = Z \frac{m_n}{\cos \beta}$

for this example $PCD = \frac{21 \times 5}{\cos 30^\circ} = 121.24 \text{ mm}$

hence $F_t = \frac{T_q \times 2}{PCD} = \frac{749.7 \times 2 \times 1000}{121.24} = 12\,367 \text{ N}$

Separating force at gear $F_r = F_t \times \frac{\tan \psi_n}{\cos \beta} = 12\,367 \times \frac{\tan 20^\circ}{\cos 30^\circ} = 5198 \text{ N}$

Axial force at gear $F_a = F_t \times \tan \beta = 12\,367 \times \tan 30^\circ = 7140 \text{ N}$

NOTE: In calculations of the reactions of the bearings, the masses of the shaft and coupling are neglected.

Reactions in the horizontal plane are calculated as follows:

- (a) Moments around floating bearing

$$400 \times R_{H2} = 200 \times 5198 + 60.6 \times 7140$$

$$\text{i.e., } R_{H2} = 3681 \text{ N}$$

- (b) Moments around fixed bearing

$$400 \times R_{H1} = 200 \times 5198 - 60.6 \times 7140$$

$$\text{i.e., } R_{H1} = 1517 \text{ N}$$

Reactions in the vertical plane are calculated as follows:

- (i) Moments around fixed bearing

$$400 \times R_{V1} = 200 \times 12\,367$$

$$\text{i.e., } R_{V1} = 6183.5 \text{ N}$$

$$R_{V1} + R_{V2} = F_t$$

$$\text{Therefore, } R_{V2} = 6183.5 \text{ N}$$

As the reaction is larger at the fixed bearing, the total reaction is calculated from its horizontal and vertical components—

$$R_2 = \sqrt{R_{V2}^2 + R_{H2}^2} = \sqrt{6183.5^2 + 3681^2} = 7196 \text{ N}$$

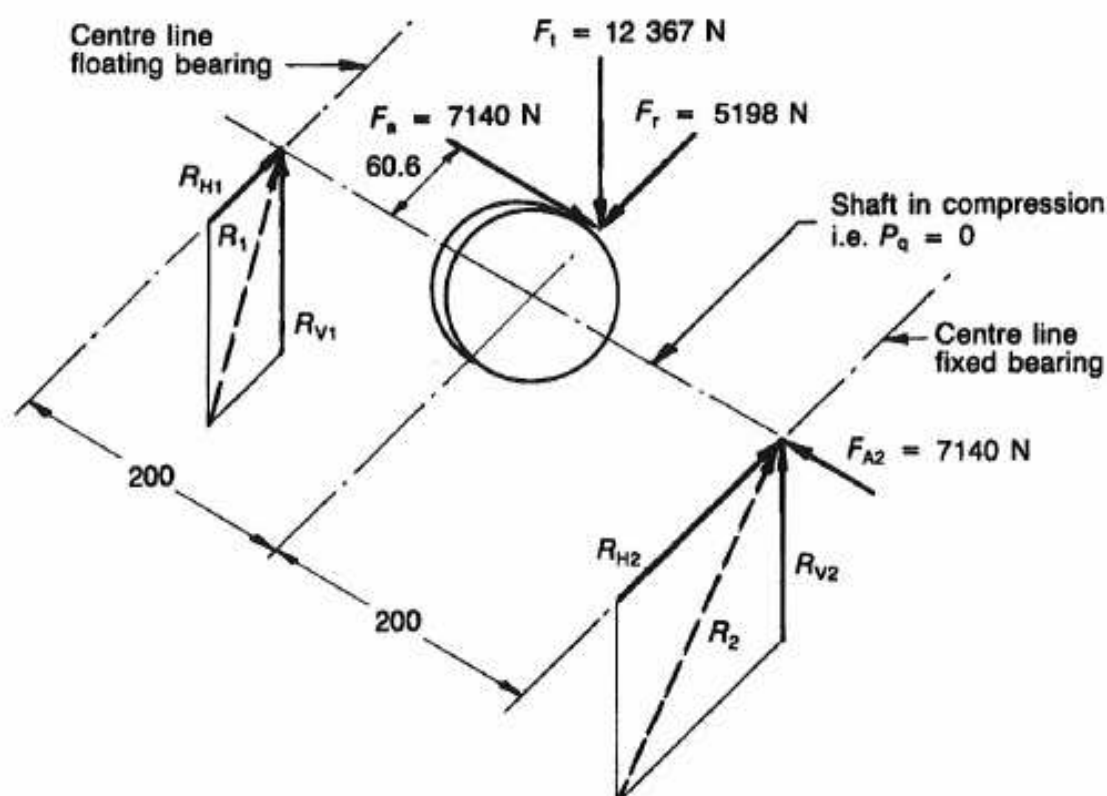


FIGURE E2 FORCES ACTING ON SHAFT 1

E3.2.5 Calculation of maximum bending moment at the centre-line of the pinion

As the pinion is equidistant from both bearings, the maximum bending moment will be—

$$M_q = R_2 \times 200 = \frac{7196 \times 200}{1000} = 1439 \text{ N.m}$$

E3.2.6 Calculation of shaft diameter

For this example, Formula 3 of Table 2 is applicable, that is—

$$D^3 = \frac{10^4 F_s}{F_R} K_s K \sqrt{\left(M_q + \frac{P_q D}{8000} \right)^2 + \frac{3}{4} T_q^2}$$

NOTE: There are no tensile forces present, so $P_q = 0$ (see Clause 7.2).

(a) Selection of various factors.

- (i) K_s is selected from Figure 1 and at this stage it is taken for the PCD of pinion 1 (= 121.4 mm).

- (ii) It is taken that $K_s = 1.7$

As the shaft geometry is not known at this stage, the various stress-raising factors (K) can be assumed only. It is assumed that the shaft will be a stepped one and will have a keyway.

From Figure 4 the selection is $K_{\text{step}} = 2.0$

From Figure 7 the selection is $K_{\text{keyway}} = 2.22$

It is also assumed that the stress-raising characteristics are separated by an axial distance not greater than $0.16D$ (see Clause 8.2(d)).

Therefore, $K = 2.22 + 2.0 \times 0.2 = 2.62$

(b) Calculation of minimum shaft diameter—

$$D^3 = \frac{10^4 \times 1.2 \times 1.7 \times 2.62}{0.45 \times 500} \sqrt{1439^2 + 0.75 \times 749.7^2}$$

from which $D = 72.1 \text{ mm}$

E3.2.7 Establishing shaft geometry

On the basis of the shaft diameter obtained in Paragraph E3.2.6(b) and the selection of bearings for the given application, the shaft geometry shown in Figure E3 has been determined.

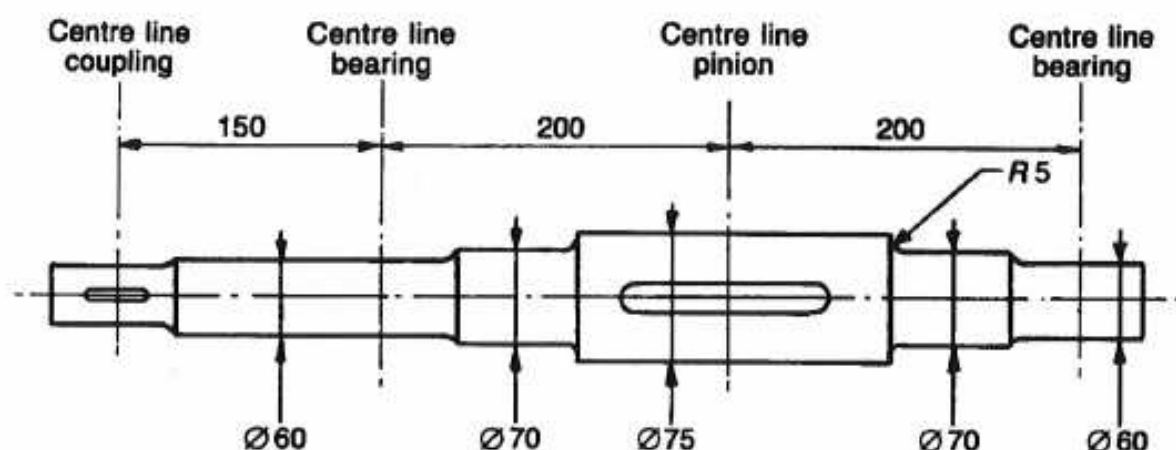


FIGURE E3 DESIGN OF SHAFT 1

E3.2.8 Checking of shaft diameter

For checking the design of the shaft, Formula 3 of Table 2 is applicable, as it was in Paragraph E3.2.6.

Selection of various factors:

(a) K_s is selected from Figure 1, for $D = 75$ mm $K_s = 1.6$.

(b) Correction factor Δ is selected from Figure 3.

$$\frac{D_1}{D} = \frac{75}{70} = 1.07, \text{ therefore } \Delta = 0.125$$

(c) Calculation of Z (see Figure 4)

$$Z = \frac{R}{D} + \Delta = \frac{5}{70} + 0.125 = 0.196$$

(d) Selection of K from Figure 4 $K_{\text{step}} = 1.3$.

(e) Selection of K from Figure 7 $K_{\text{keyway}} = 2.22 \times 1.1 = 2.44$.

It is assumed, that the stress-raising characteristics are separated by an axial distance not greater than $0.16D$ (see Clause 8.2(d)).

Therefore, $K = 2.44 + 1.3 \times 0.2 = 2.7$

E3.2.9 Calculation/checking of minimum shaft diameter.

$$D^3 = \frac{10^4 \times 1.2 \times 1.6 \times 2.7}{0.45 \times 500} \sqrt{1439^2 + 0.75 \times 749.7^2}$$

from which $D = 71.4$ mm

Thus the diameter of 75 mm shown in Figure E3 at the centre-line of the pinion will be satisfactory from strength consideration.

E3.3 Analysis while lowering the maximum load and applying the maximum braking torque

E3.3.1 Calculations of angular acceleration of the driving medium

It is assumed that the load is being lowered at the same speed with which it is hoisted, i.e., 0.168 m/s (= 10 m/min).

Therefore, for braking $T_M = 1000 \text{ N.m}$

The formula for the output torque of shaft 4 in Table 1 may be simplified to—

$$T_{4 \text{ out}} = \left[(T_M - I_{E1}\alpha) \frac{1}{\eta_{1/2}\eta_{2/3}\eta_{3/4}} - I_{E2}\alpha \frac{1}{\eta_{2/3}\eta_{3/4}} - I_{E3}\alpha \frac{1}{\eta_{3/4}} - I_{E4}\alpha \right] \frac{N}{N_4}$$

It is known from Paragraph E3.2.2 that the torque required to raise the load (or holding it in balance) is T_{DRUM} .

By equating the required torque and the output torque on shaft 4, i.e. $T_{4 \text{ out}} = T_{\text{DRUM}}$, we obtain the following:

$$\left[(1000 - 1.298\alpha) \frac{1}{0.96^3} - 0.062\alpha \frac{1}{0.96^2} - 0.048\alpha \frac{1}{0.96} - 0.506\alpha \right] \frac{24}{0.32} = 41120$$

$$\text{i.e., } 1130.3 - 1.467\alpha - 0.067\alpha - 0.050\alpha - 0.506\alpha = 548.3$$

$$\text{From which we obtain } \alpha = \frac{582.0}{2.090} = 278.5 \text{ rad/s}^2$$

E3.3.2 Calculation of the output torque for shaft 1

$$T_{1 \text{ out}} = T_M - I_{E1}\alpha = 1000 - 1.298 \times 278.5 = 638.5 \text{ N.m (= } T_q)$$

E3.3.3 Calculation of forces arising at the gear and reactions arising at the bearings for shaft 1

(see also basic formulas in Paragraph E3.2.4)

NOTE: In this case, changing the direction of rotation does not reverse the axial load in the shaft.

$$T_q = 638.5 \text{ N.m}$$

$$F_t = \frac{638.5 \times 2 \times 1000}{121.24} = 10\,533 \text{ N}$$

$$F_r = F_t \times \frac{\tan \psi_n}{\cos \beta} = 10\,533 \times \frac{\tan 20^\circ}{\cos 30^\circ} = 4427 \text{ N}$$

$$F_a = F_t \times \tan \beta = 10\,533 \times \tan 30^\circ = 6081 \text{ N}$$

$$R_{H2} = \frac{200 \times 4427 + 60.6 \times 6081}{400} = 3135 \text{ N}$$

$$R_{H1} = 1292 \text{ N}$$

$$R_{V1} = R_{V2} = \frac{10\,533}{2} = 5267 \text{ N}$$

As the reaction is larger at the fixed bearing, the total reaction is calculated as—

$$R_1 = \sqrt{R_{V1}^2 + R_{H2}^2} = \sqrt{5267^2 + 3135^2} = 6129 \text{ N}$$

E3.3.4 *Calculation of maximum bending moment at the centreline of the pinion*

(see also Paragraph E3.2.5)

$$M_q = R_1 \times 200 = \frac{6129 \times 200}{1000} = 1226 \text{ N.m}$$

NOTE: As both M_q and T_q in this case (lowering the load) are less than in the previous case (raising the load), the diameter of 75 mm shown in Figure E3 at the centre-line of the pinion will be again satisfactory from strength considerations.

APPENDIX F

TYPICAL WORKED EXAMPLE—CONVEYOR DRIVE

(Informative)

F1 SCOPE

This Appendix illustrates the application of this Standard for the determination of the minimum diameter of a drive pulley shaft required for a reversing conveyor.

F2 DATA

F2.1 General

A reversing conveyor with a 1400 mm wide SR 2500 steel cord belt, 2000 m between terminal pulleys, four independent drives each one connected to a shaft extension on the two driving pulleys. Each drive consists of a 370 kW squirrel cage electric motor driving a fluid coupling connected to a two-stage hollow output shaft gear reducer. These components are bolted to a drive base that is supported at the motor end by a pivoted arm and the other end by the drive pulley shaft extensions, which fit into the gearbox hollow output shaft. (see Figure F1).

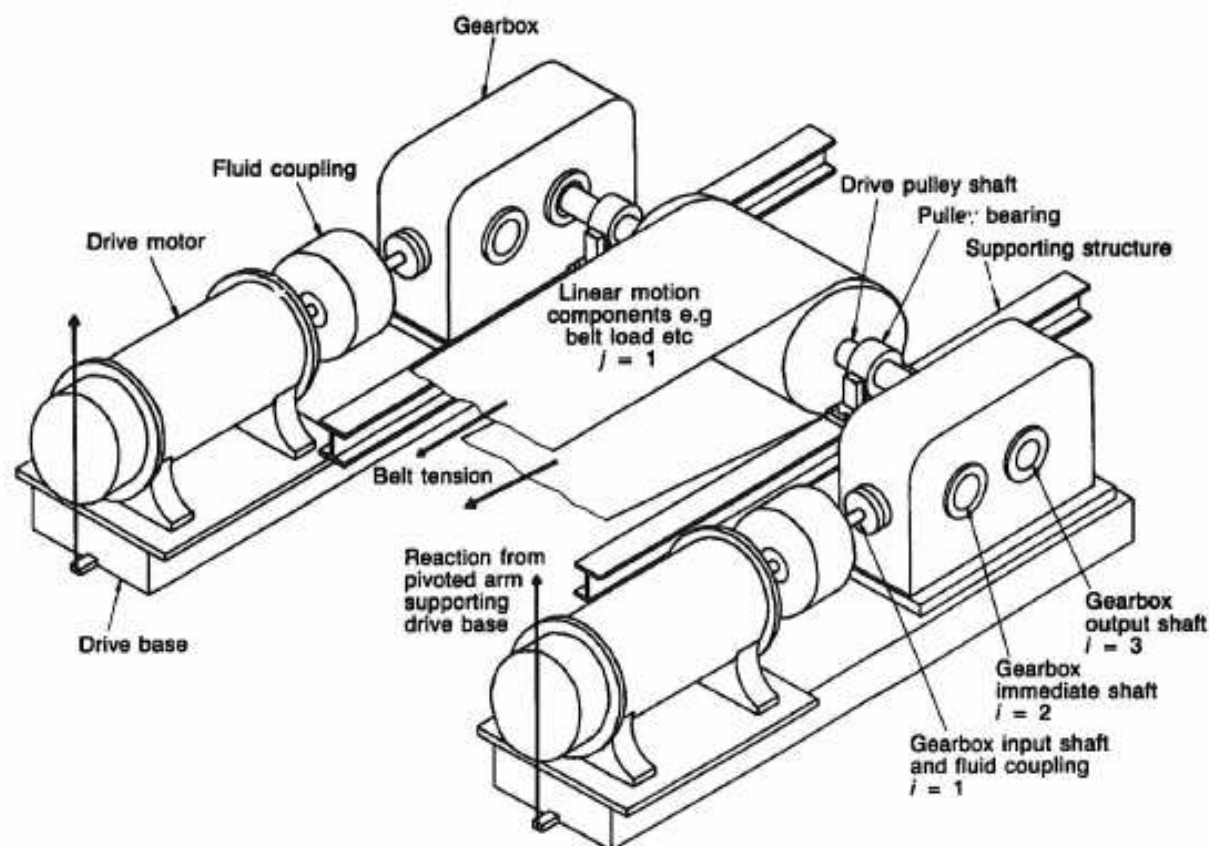


FIGURE F1 DIAGRAM OF CONVEYOR DRIVE

F2.2 Data available to carry out subsequent calculations

- (a) Assume 3% slip in fluid coupling and full load output speed of 370 kW drive motor is 970 r/min.

Therefore

$$N = \frac{970}{60} = 16.17 \text{ r/s}$$

$$\text{and } N_1 = 0.97 \times 16.17 = 15.7 \text{ r/s}$$

- (b) Gearbox reduction ratio is 10:1, i.e. $\frac{N_1}{N_3} = 10$, and it is assumed that

$$\frac{N_1}{N_2} = \frac{N_2}{N_3}$$

Therefore,

$$\frac{N_1}{N_2} = \frac{N_2}{N_3} = \sqrt{10}$$

and $N_2 = 4.96$ r/s and $N_3 = 1.57$ r/s

- (c) Gearbox drive efficiencies.

First reduction (helical bevel wheel and pinion) $\eta_{1/2} = 0.95$

Second reduction (helical wheel and pinion) $\eta_{2/3} = 0.98$

- (d) Torque applied to the mechanism during starting (fluid coupling output torque is assumed to be limited to 1.5 times rated (full load) motor torque).

Therefore,

$$T_M = 1.5 \frac{370 \times 1000}{2\pi \times 16.17} = 5463 \text{ N.m}$$

- (e) Determination of rotational mass moments of inertia of component shafts and their components (when present).

- (i) First shaft

$$I_{E1} = I_1 \left(\frac{N_1}{N} \right)^2 = 0.5 \left(\frac{15.7}{16.17} \right)^2 = 0.47 \text{ kg.m}^2$$

where I_1 = inertia of gearbox input shaft (fluid coupling half and bevel pinion and shafting) = 0.5 kg.m^2

- (ii) Second shaft

$$I_{E2} = I_2 \left(\frac{N_2}{N} \right)^2 = 10 \left(\frac{4.96}{16.17} \right)^2 = 0.94 \text{ kg.m}^2$$

where I_2 = inertia of gearbox intermediate shaft (bevel gear helical pinion and shaft) = 10 kg.m^2

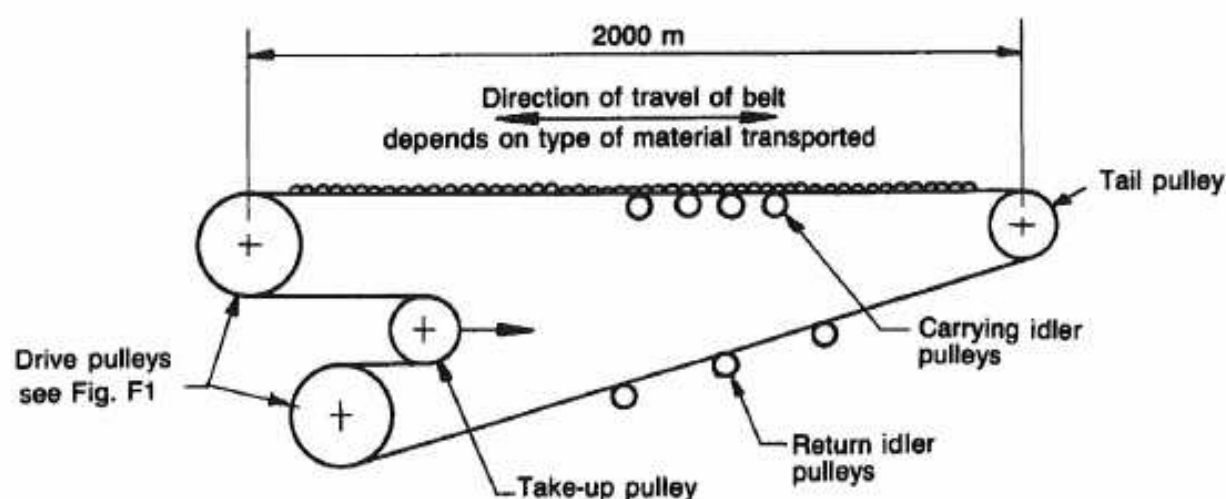


FIGURE F2 DIAGRAM OF CONVEYOR

Drive pulleys (each)	Moment of inertia (including that of gearbox output shaft and helical wheel)—460 kg.m ² Effective diameter 1070 mm (=D _{DP})
Take up pulley	Moment of inertia 135 kg.m ² Effective diameter 915 mm (=D _{TP})
Tail pulley	Moment of inertia 135 kg.m ² Effective diameter 915 mm (=D _{TP})
Carrying idler pulleys (each)	Moment of inertia 0.16 kg.m ² Effective diameter 152.4 mm (=D _{CIP}) Pitch between rollers 1200 mm
Return idler pulleys (each)	Moment of inertia 0.1 kg.m ² Effective diameter 158.7 mm (=D _{RIP}) Pitch between rollers 3000 mm

NOTE: Effective diameter = Pulley outside dia. + the belt carcass thickness + twice the applicable cover rubber thickness.

Mass of material carried by belt—271 kg/m

Mass of belt—56 kg/m

$$I_{E3} = I_{R3} + I_{L1}$$

The rotational speeds of the various pulleys are determined as follow:

$$\text{Take-up pulley and tail pulley} \quad N_{TP} = N_3 \times \frac{D_{DP}}{D_{TP}} = 1.57 \times \frac{1070}{915} = 1.84 \text{ r/s}$$

$$\text{Carrying idler pulley} \quad N_{CIP} = N_3 \times \frac{D_{DP}}{D_{CIP}} = 1.57 \times \frac{1070}{152.4} = 11.02 \text{ r/s}$$

$$\text{Return idler pulley} \quad N_{RIP} = N_3 \times \frac{D_{DP}}{D_{RIP}} = 1.57 \times \frac{1070}{158.7} = 10.59 \text{ r/s}$$

$$I_{R3} = 2 \times 460 \times \left(\frac{1.57}{16.17} \right)^2 + 135 \times \left(\frac{1.84}{16.17} \right)^2 + 135 \times \left(\frac{1.84}{16.17} \right)^2 + 0.16 \times \frac{2000}{1.2} \times \left(\frac{11.02}{16.17} \right)^2 + 0.1 \times \frac{2000}{3.0} \times \left(\frac{10.59}{16.17} \right)^2 = 164.62 \text{ kg.m}^2$$

$$I_{L1} = 2000 \times (271 + 2 \times 56) \times \left(\frac{\pi \times 1.070 \times 1.57}{2\pi \times 16.17} \right)^2 = 2066.88 \text{ kg.m}^2$$

$$I_{E3} = 164.62 + 2066.88 = 2231.5 \text{ kg.m}^2$$

NOTE: The total inertia and efficiency of the gearbox are often stated in catalogues for 'ex-stock' gearboxes.

F3 CALCULATIONS

F3.1 Calculation of system acceleration

It is assumed, that the resisting torque of the conveyor at full speed equals the rated torque capacity of the drives at the third shaft.

Therefore, the resisting torque for each drive will be, due to the motion of the conveyor

$$\frac{T_M}{1.5} \frac{N}{N_3} \eta_{1/2} \eta_{2/3}$$

By equating torque output to resisting torque at shaft 3 we obtain the following equation:

$$\left[(T_M - I_{E1} \alpha) \frac{N}{N_2} \eta_{1/2} - I_{E2} \frac{N}{N_2} \alpha \right] \frac{N_2}{N_3} \eta_{2/3} = \frac{I_{E3}}{4} \frac{N}{N_3} \alpha + \frac{T_M}{1.5} \frac{N}{N_3} \eta_{1/2} \eta_{2/3}$$

Solving the above equation for α , we receive:

$$\alpha = \frac{\frac{T_M}{3} \eta_{1/2} \eta_{2/3}}{I_{E1} \eta_{1/2} \eta_{2/3} + I_{E2} \eta_{2/3} + \frac{I_{E3}}{4}} = \frac{\frac{5463}{3} \times 0.95 \times 0.98}{0.47 \times 0.95 \times 0.98 + 0.94 \times 0.98 + \frac{2231.5}{4}} = 3.032 \text{ rad/s}^2$$

F3.2 Calculation of torque on pulley shaft

When starting, the torque input to shaft 3 equals (see Table 1)

$$\begin{aligned} T_{3in} &= \left[(T_M - I_{E1} \alpha) \frac{N}{N_2} \eta_{1/2} - I_{E2} \frac{N}{N_2} \alpha \right] \frac{N_2}{N_3} \eta_{2/3} = \\ &= \left[(5463 - 0.47 \times 3.032) \frac{16.17}{4.96} \times 0.95 - 0.94 \times 3.032 \times \frac{16.17}{4.96} \right] \frac{4.96}{1.57} \times 0.98 = 52\,341 \text{ N.m} \end{aligned}$$

F3.3 Calculation of forces acting on pulley shaft due to belt tensions

Drive pulley belt tensions arise during transmission of the torque from the drives. It is assumed that belt tensions act horizontally.

As the torque due to belt tensions equals to the torque due to the drives (see Figure F3) the following equation can be established:

$$\frac{1.070}{2} \left(\frac{BT_1}{2} - \frac{BT_2}{2} \right) = \frac{1.070}{2} \left(\frac{BT_2}{2} - \frac{BT_3}{2} \right) = 52\,341 \text{ N.m, hence}$$

$$(BT_1 - BT_2) = (BT_2 - BT_3) = 195\,667 \text{ N}$$

where BT = belt tension

According to conveyor design manuals and/or belt manufacturers' catalogues

$$BT_3 = \frac{1}{2.178^{\mu\theta} - 1} (BT_1 - BT_2)$$

where

$\mu = 0.3$, coefficient of friction between belt and drive pulley

$\theta = \pi(\text{rad})$, angle of wrap of belt around drive pulley.

Therefore,

$$BT_3 = \frac{195\,667}{2.178^{0.3 \times \pi} - 1} = 124\,920 \text{ N}$$

Consequently,

$$BT_2 = 124\,920 + 195\,667 = 320\,587 \text{ N}$$

and

$$BT_1 = 320\,587 + 195\,667 = 516\,254 \text{ N}$$

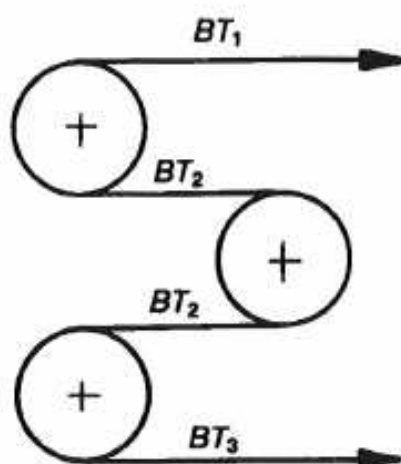


FIGURE F3 BELT TENSION DIAGRAM

F3.4 Calculation of force on drive pulley shaft extension due to drive (see Figure F4)

All moments are taken about the drive rear support to determine gearbox reaction on shaft extension during starting.

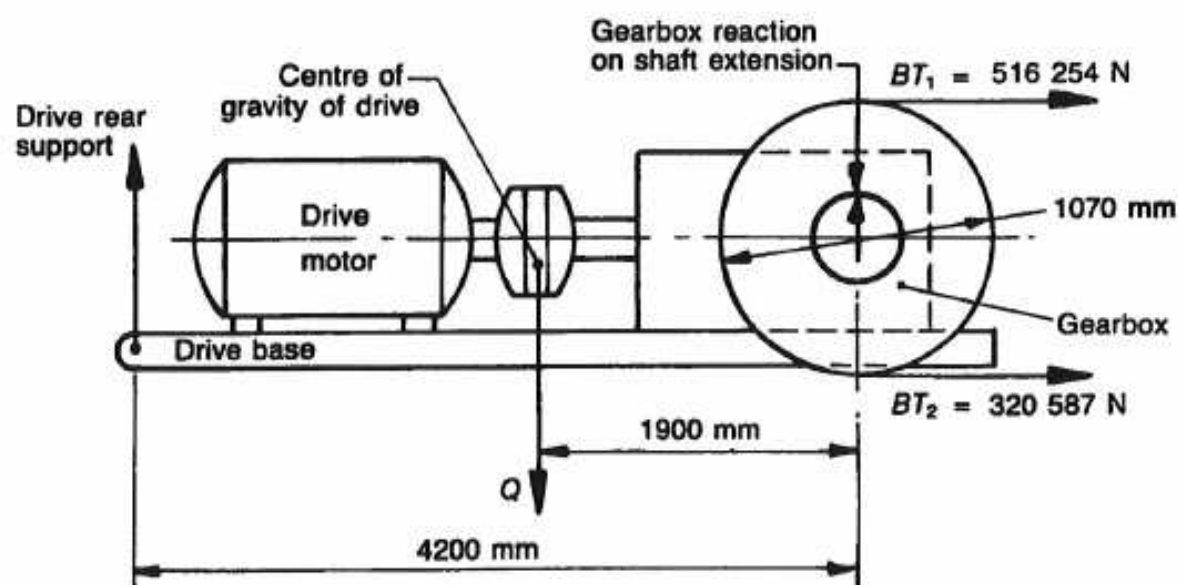
Gearbox reaction on shaft extension

$$= \frac{Q \times 9.81(4200 - 1900) \pm \left(\frac{BT_1}{2} - \frac{BT_2}{2} \right) \frac{1070}{2}}{4200}$$

$$= \frac{12\,500 \times 9.81(4200 - 1900) + (258\,127 - 160\,293.5) \times 535}{4200} = 79\,614 \text{ N}$$

NOTES:

- 1 Sign (+ or -) before belt tension component depends on direction of travel.
- 2 For all forces and bending moments acting on the drive pulley shaft, see Figure F5.



Q = mass of drive unit (= 12 500 kg)

FIGURE F4 DIAGRAM OF FORCES ACTING ON PULLEY SHAFT DUE TO DRIVE

F3.5 Selection of formula for calculating shaft diameter

As the number of starts in a year is over 600 and the number of revolutions of shaft 3 is over 900 and there are power-applied torque reversals, Formula 3 is to be used to determine the diameter of the pulley shaft.

$$D^3 = \frac{10^4 \times F_s K_s K}{F_R} \sqrt{\left(M_q + \frac{P_q D}{8000} \right)^2 + \frac{3}{4} T_q^2}$$

NOTE: For calculated values of D , see Table F1.

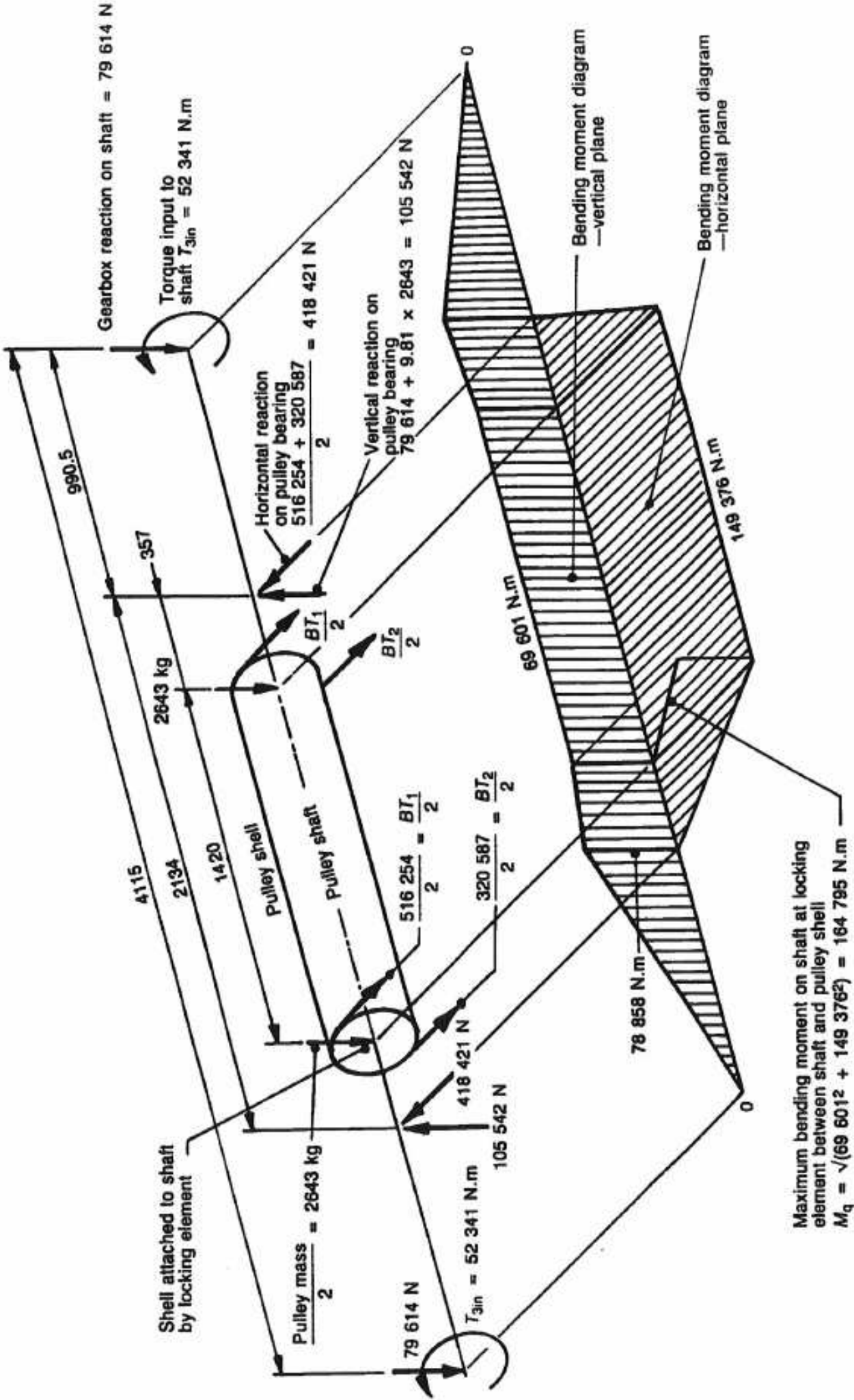


FIGURE F5 FORCE AND BENDING MOMENT DIAGRAM FOR PULLEY SHAFT

TABLE F1
CALCULATION OF *D*

Case 1				Case 2			
Minimum diameter <i>D</i> required at pulley bearings				Minimum diameter <i>D</i> required at locking element between pulley shell and shaft			
Symbol	Value	Remarks		Symbol	Value	Remarks	
F_U	410 MPa	Specified minimum tensile strength for shaft material		F_U	410 MPa	Specified minimum tensile strength for shaft material	
F_R	0.45×410 = 185 MPa			F_R	0.45×410 = 185 MPa		
K_S	1.8	See Figure 1; correct if necessary after each iteration		K_S	1.8	See Figure 1; correct after each iteration	
K	1.5	See Figure 5		K	1.3	Obtained from consultation with locking element supplier	
M_q	78 858 N.m	See Figure F5 for value		M_q	164 795 N.m	See Figure F5 for value	
P_q	0			P_q	0		
T_q	52 341 N.m	See Figure F5 for value		T_q	52 341 N.m	See Figure F5 for value	
D	251.6 mm	Substitute above values for Formula 3 and solve for <i>D</i>		D	296.0 mm	Substitute above values for Formula 3 and solve for <i>D</i>	

NOTES:

- 1 The actual diameter *D* of shaft will depend on the size of pulley bearing and locking assembly selected to accord with the manufacturer's requirements.
- 2 The designer may be required to check the diameters at other cross-sections, e.g., at steps for bearings.

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